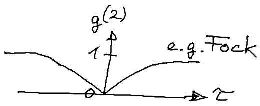
English Summary:

Conditions for nonclassical light



$g^{(2)}(\tau) > g^{(2)}(0)$, $g^{(2)}(0) < 1$ (sub-Poissonian correlations)

4.6 Quantum Master equation

$$\varrho_{SR}(t) = \underset{\text{system}}{\varrho_S(t)} \otimes \underset{\text{reservoir}}{\varrho_R(0)} + \underset{\text{coupling}}{\varrho_c(t)} \quad , \quad \varrho_S(t) = \mathrm{tr}_R\, \varrho_{SR} \text{ reduced density matrix}$$

$$\boxed{\dot{\varrho}_S(t) = -\frac{1}{\hbar^2}\, \mathrm{tr}_R \int_0^t dt' \left[V(t), \left[V(t'), \varrho_S(t)\right] \otimes \varrho_R(0)\right]}$$

Master eq. in interaction picture; interaction Ham. V; Born-Markov approx.

Beispiel: gedämpfter harmon. Oszillator

Ham. op. des System $S \otimes R$: $\quad H = \underbrace{H_S + H_R}_{H_0} + \underbrace{H_{SR}}_{V}$

$H_S = \hbar\omega_0 a^+ a$ harm. Osz. $[a, a^+] = 1$

$H_R = \sum_j \hbar\omega_j r_j^+ r_j$ bosonische Reservoirmoden $[r_j, r_i^+] = \delta_{ij}$

im therm. Gl. $\quad \varrho_R = \prod_j e^{-\frac{\hbar\omega_j r_j^+ r_j}{kT}} \left(1 - e^{-\frac{\hbar\omega_j}{kT}}\right)$

(z.B. Vakuum-Strahlungsfeld oder Phononen-Moden in Festkörpern)

↑ Zerfall einer Mode ω_0 in einer opt. Kavität

↑ Zerfall eines angeregten Atoms durch spontane Emission

$$H_{SR} = \sum_j \hbar\left(\kappa_j^* a r_j^+ + \kappa_j a^+ r_j\right) \qquad \kappa_j \in \mathbb{C} \quad \text{Dämpfungsrate}$$

$$= \hbar\left(a\Gamma^+ + a^+\Gamma\right) \qquad \Gamma, \Gamma^+ \quad \text{Kopplungsop.}$$

im WW-Bild:

$$a_W(t) = e^{i\omega_0 a^+ a t} \underset{\text{Schröd. Bild}}{a}\, e^{-i\omega_0 a^+ a t} \overset{(*)}{=} a e^{-i\omega_0 t}$$

$$a_W^+(t) = e^{i\omega_0 a^+ a t} a^+ e^{-i\omega_0 a^+ a t} = a^+ e^{i\omega_0 t}$$

⊛ denn $\dot a = -i\omega_0\,[a, a^+ a] = -i\omega_0\,(a a^+ a - a^+ a a) = -i\omega_0 a$

(Heisenberg-Gl.) $\qquad [a, a^+] = 1$

$$\Gamma_W^+(t) = e^{i\sum_n \omega_n r_n^+ r_n t} \sum_j \kappa_j^* r_j^+ \, e^{-i\sum_m \omega_m r_m^+ r_m t} = \sum_j \kappa_j^* r_j^+ e^{i\omega_j t} \quad \Rightarrow \langle \Gamma_W^+(t)\rangle_R = 0$$

$$\Gamma_W(t) = e^{i\sum_n \dots} \sum_j \kappa_j r_j \, e^{-i\sum_m \dots} = \sum_j \kappa_j r_j e^{-i\omega_j t} \quad \Rightarrow \langle \Gamma_W(t)\rangle_R = 0$$

<u>Mastergl. (Born-Markov) im WW-Bild</u>

$\langle r_n r_m\rangle = \langle r_n^+ r_m^+\rangle = 0$

$\langle r_n^+ r_m\rangle = \bar n_n \delta_{nm}$

$\langle r_n r_m^+\rangle = (\bar n_n + 1)\delta_{nm}$

$$\hat\varrho_S = -\int_0^t dt' \Big\{ (a a \varrho_S(t) - a \varrho_S(t) a)\, e^{-i\omega_0(t+t')} \underbrace{\langle \Gamma_W^+(t)\Gamma_W^+(t')\rangle_R}_{0}$$

in Fock-Basis nachrechnen

$$+ (a^+ a^+ \varrho_S(t) - a^+ \varrho_S(t) a^+)\, e^{i\omega_0(t+t')} \underbrace{\langle \Gamma_W(t)\Gamma_W(t')\rangle_R}_{0}$$

$$+ (a a^+ \varrho_S(t) - a^+ \varrho_S(t) a)\, e^{-i\omega_0(t-t')} \underbrace{\langle \Gamma_W^+(t)\Gamma_W(t')\rangle_R}_{\sum_j |\kappa_j|^2 e^{i\omega_j(t-t')}\, \bar n(\omega_j, T)}$$

$$+ (a^+ a \varrho_S(t) - a \varrho_S(t) a^+)\, e^{i\omega_0(t-t')} \underbrace{\langle \Gamma_W(t)\Gamma_W^+(t')\rangle_R}_{\sum_j |\kappa_j|^2 e^{-i\omega_j(t-t')}\, (\bar n(\omega_j, T) + 1)}$$

$$+ h.c. \Big\}$$

mit $\bar n(\omega_j, T) = \mathrm{tr}_R\,(\varrho_R r_j^+ r_j) = \dfrac{e^{-\frac{\hbar\omega_j}{kT}}}{1 - e^{-\frac{\hbar\omega_j}{kT}}}$ Bose-Verteilung

$\tau = t - t'$

$$\dot\varrho_S = -(a a^+ \varrho_S - a^+ \varrho_S a) \int_0^t d\tau\, e^{-i\omega_0\tau} \langle \Gamma_W^+(t)\Gamma_W(t-\tau)\rangle_R + h.c.$$

$$- (a^+ a \varrho_S - a \varrho_S a^+) \int_0^t d\tau\, e^{i\omega_0\tau} \langle \Gamma_W(t)\Gamma_W^+(t-\tau)\rangle_R + h.c.$$

Kontinuumslimes der Reservoir-Korrelationsfkt.en:

$$\langle \Gamma_W^+(t)\Gamma_W(t-\tau)\rangle_R = \int_0^\infty d\omega\, e^{i\omega\tau} g(\omega)\, |\kappa(\omega)|^2\, \bar n(\omega, T) \approx \delta(\tau)$$

Zustandsdichte des Reservoirs

$$\langle \Gamma_W(t)\Gamma_W^+(t-\tau)\rangle_R = \int_0^\infty d\omega\, \underbrace{e^{-i\omega\tau}}_{\text{schnell oszill.}} \underbrace{g(\omega)\, |\kappa(\omega)|^2\, [\bar n(\omega, T) + 1]}_{\text{langsam variabel}} \approx \delta(\tau)$$

$\int \approx 0$ für $\tau \gg t_R = \dfrac{\hbar}{kT}$ Reservoir-Korrelationszeit

$$\Longleftrightarrow \quad \omega\tau = \frac{\hbar\omega}{kT} \gg 1$$

Def. Zerfallsraten:

$$\gamma_1 := \int_0^t d\tau \int_0^\infty d\omega\, e^{-i(\omega-\omega_0)\tau}\, g(\omega)\,|K(\omega)|^2$$

$$\gamma_2 := \int_0^t d\tau \int_0^\infty d\omega\, e^{-i(\omega-\omega_0)\tau}\, g(\omega)\,|K(\omega)|^2\, \bar n(\omega, T)$$

Hauptbeitrag der τ-Integration: kleine τ $\Rightarrow$ ersetze $\int_0^t \rightarrow \int_0^\infty d\tau$

$$\lim_{t\to\infty} \int_0^t d\tau\, e^{-i(\omega-\omega_0)\tau} = \pi\delta(\omega-\omega_0) + i\frac{P}{\omega_0-\omega}$$

Cauchy-Hauptwert

$$\Rightarrow \quad \gamma_1 = \frac{\gamma}{2} + i\Delta$$

$$\gamma_2 = \frac{\gamma}{2}\bar n + i\Delta'$$

mit $\gamma := 2\pi g(\omega_0)\,|K(\omega_0)|^2$

$$\Delta := P\int_0^\infty d\omega\, \frac{g(\omega)\,|K(\omega)|^2}{\omega_0-\omega}$$

$$\Delta' := P\int_0^\infty d\omega\, \frac{g(\omega)\,|K(\omega)|^2}{\omega_0-\omega}\,\bar n$$

Mastergl.

$$\dot\rho_s = (a^+\rho_s a - aa^+\rho_s)\gamma_2^* + \overbrace{(a^+\rho_s a - \rho_s aa^+)\gamma_2}^{h.c. \qquad \rho_s=\rho_s^+}$$

$$+ (a\rho_s a^+ - a^+a\rho_s)(\gamma_1+\gamma_2) + \underbrace{(a\rho_s a^+ - \rho_s a^+a)(\gamma_1^*+\gamma_2^*)}_{h.c.}$$

$$= a^+\rho_s a\,\gamma\bar n - (aa^+\rho_s + \rho_s aa^+)\frac{\gamma}{2}\bar n + \underbrace{(aa^+\rho_s - \rho_s aa^+)}_{a^+a\rho_s - \rho_s a^+a} i\cancel{\Delta'} \qquad [a, a^+] = 1$$

$$+ a\rho_s a^+\gamma(1+\bar n) - (a^+a\rho_s + \rho_s a^+a)\frac{\gamma}{2}(1+\bar n) - (a^+a\rho_s - \rho_s a^+a)\, i(\Delta + \cancel{\Delta'})$$

$$= -i\Delta\,[a^+a, \rho_s] + \frac{\gamma}{2}(2a\rho_s a^+ - a^+a\rho_s - \rho_s a^+a)$$

$$+ \gamma\bar n(a\rho_s a^+ + a^+\rho_s a - a^+a\rho_s - \rho_s aa^+)$$

Rücktransform. WW-Bild $\rightarrow$ Schrödinger-Bild $\rho_s^{(S)}$

$$\dot\rho_s^{(S)} = -\frac{i}{\hbar}[H_S, \rho_S^{(S)}] + e^{-\frac{i}{\hbar}H_S t}\,\dot\rho_s^{(S)}\, e^{\frac{i}{\hbar}H_S t}$$

$$e^{-i\omega_0 a^+ a t}\,\underbrace{a\rho_S a^+}_{\text{WW-Bild}}\,e^{i\omega_0 a^+ a t} = \underbrace{a\rho_S^{(S)} a^+}_{\text{Schrödinger-Bild}} \qquad H_S = \hbar\omega_0 a^+ a$$

Mastergl. für gedämpften harmon. Osz.:
(Schrödinger-Bild)

$$\dot\rho_S^{(S)} = -i\underbrace{\omega_0'}_{\omega_0+\Delta}\,[a^+a, \rho_S^{(S)}] + \frac{\gamma}{2}\left(2a\rho_S^{(S)}a^+ - a^+a\rho_S^{(S)} - \rho_S^{(S)}a^+a\right)$$

$$+\,\gamma\bar n\left(a\rho_S^{(S)}a^+ + a^+\rho_S^{(S)}a - a^+a\rho_S^{(S)} - \rho_S^{(S)}aa^+\right)$$

renormierte Frequenz durch Ankoppl. ans Bad

reversible Dyn.

dissipative Dynamik

Dämpfung durch
- spontane Em.
- stimul. Em. / Abs. ($\sim \bar n$) von Bad-Bosonen

(z.B. Photonen, Phononen)

$$\Leftrightarrow \dot\rho_S^{(S)} = -i\omega_0'[a^+a, \rho_S^{(S)}] + \frac{\gamma}{2}\left([a, \rho_S^{(S)}a^+] + [a\rho_S^{(S)}, a^+]\right) + \gamma\bar n\left([a\rho_S^{(S)}, a^+] + [a^+, \rho_S^{(S)}a]\right)$$

$\rho a a^+ = \rho a^+ a + \rho$
$a\rho a^+ = a a^+ \rho$
$= a^+ a \rho + \rho$
$= a^+ \rho a + \rho$

$$\Leftrightarrow \boxed{\dot\rho_S^{(S)} = -i\omega_0'[a^+a, \rho_S^{(S)}] + \frac{\gamma}{2}(\bar n+1)\Big(\underbrace{2a\rho_S^{(S)}a^+}_{n+1\to n} - \underbrace{\{a^+a, \rho_S^{(S)}\}}_{\substack{\text{Anti-Kommutator}\\ n\to n-1}}\Big) + \frac{\gamma}{2}\bar n\Big(\underbrace{2a^+\rho_S^{(S)}a}_{n-1\to n} - \underbrace{\{aa^+, \rho_S^{(S)}\}}_{n\to n+1}\Big)}$$

Lindblad-Form