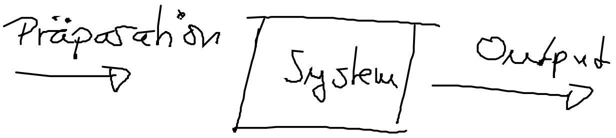
Biexziton-Kaskade II:

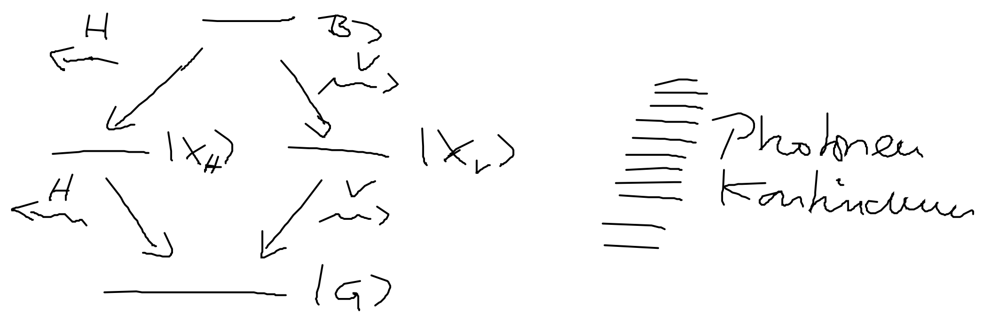
Richard Feynman: „Nobody understands quantum theory." (1967)



$$|\psi\rangle = \frac{1}{\sqrt{2}}\left\{|HH\rangle + |VV\rangle\right\}$$

Messbar: niemals die Wellenfunktion, nur die Dichtematrixelemente

$$\rho = \sum_{ij=H,V} c_{ij}\, |ij\rangle\langle ij|$$



$$H = \int dk\, g_0 \Big\{ e^{i(\omega_B - \omega_{XH} - \omega_k)t}\, b_{kH}\, B^+ X_H + h.a.$$
$$+ e^{i(\omega_{XH} - \omega_k)t}\, a_{kH}\, X_H^+ G + h.a.$$
$$+ \ldots \{X_H \text{ in } X_V \text{ ersetzen}\} \Big\}$$

$$|\psi\rangle = c_B\, |B\rangle + \int dk \{ c_{Hk}\, |X_H, \underbrace{1_{kH}}_{\text{Biexziton}}\rangle + c_{Vk}\, |X_V, 1_{kV}\rangle \}$$

Ansatz

$$+ \int dk \int dk' \{ c_{Hkk'}\, |G, 1_{kH} \ldots 1_{k'H}\rangle$$

$+ \langle v z r' | G, 1_{r_V} \ldots 1_{r'_V} \rangle \}$

für den Fall:

$c_B(0) = 1$, ungetriebenes System

Beschränkung auf Zwei-Anregungen

es soll gelten: $a_{rH} | G, 1_{rH} \ldots 1_{r'H} \rangle \neq 0$

Schreibweise $a_{rH} | X_H, 1_{rH} \rangle = G_{rH} b^+_{rH} | X_H, 1 \rangle$

$= 0$

$b_{rH} | X_H, 1_{rH} \rangle \neq 0$

$$i\hbar \partial_t | \Psi \rangle = H | \Psi \rangle$$

$$g^r_{BH} = g_0 \, e^{i(\omega_B - \omega_{XH} - \omega_r)t}$$

$$g^r_{XH} = g_0 \, e^{i(\omega_{XH} - \omega_r)t}$$

(dasselbe für V)

$$i\hbar \langle B | \dot{\Psi} \rangle = \langle B | H | \Psi \rangle = \int dr \, \{ g^r_{BH} c_{rH} + g^r_{BV} c_{rV} \}$$

$$\langle X_H, 1_{r_1 H} | \dot{\Psi} \rangle = -i \, g^{r_1}_{BH} c_B - i \int dr \, g^r_{XH} c_{H r r_1}$$

$$\langle G, 1_{r_1 H} 1_{r_2 H} | \dot{\Psi} \rangle = \dot{c}_{H r_1 r_2} = -i \, g^{r_2 *}_{XH} c_{H r_1}$$

(dasselbe alles für V)

Formal integrieren $c_{H r_1 r_2}(0) = 0 = c_{H r_1}(0)$

$$c_{H r_1 r_2}(t) = -i \int_0^t dt' \, g^{r_2 *}_{XH}(t') \, c_{H r_1}(t')$$

$$\dot{c}_{Hk_1} = -i\, g_{BH}^{k_1 *}(t)\, c_B(t) \; + \; (-i)^2 \int dk\, g_{XH}^{k}(t) \int_0^t dt'\, g_{XH}^{k*}(t')\, c_{Hk_1}(t')$$

$$= -i\, g_{BH}^{k_1 *}(t)\, c_B(t) - \int_0^t dt'\, c_{Hk_1}(t')\, g_0^2 \underbrace{\int dk\, e^{-i(\omega_{XH}-\omega_k)(t-t')}}_{=\frac{2\pi}{c}\delta(t-t')\, e^{-i\omega_{XH}(t-t')}}$$

$$\dot{c}_{Hk_1} = -\Gamma_H\, c_{Hk_1}(t) - i\, g_{BH}^{k_1 *}(t)\, c_B(t)$$

$$c_{Hk_1}(t) = e^{-\Gamma_H t} \int_0^t dt'\, (-i)\, g_{BH}^{k_1 *}(t')\, c_B(t')$$

einsetzen DGL $c_B(t)$

$$\dot{c}_B = -\int dk\, g_{BH}^{k}(t)\, e^{-\Gamma t} \int_0^t dt'\, g_{BH}^{k*}(t')\, c_B(t')\, e^{\Gamma t'}$$

$$+ \{\text{dasselbe für } V\}$$

$$= -\int_0^t dt\, g_0^2\, e^{-\Gamma(t-t')}\, c_B(t') \int dk\, e^{i(\omega_B-\omega_{XH}-\omega_k)(t-t')}$$

$$= -\Gamma_H\, c_B(t) - \Gamma_V\, c_B(t)$$

$$c_B(t) = e^{-(\Gamma_H+\Gamma_V)t}\, \underbrace{c_B(0)}_{=1}$$

$$c_{H k_1}(t) = e^{-\Gamma_H t} \int_0^t dt'\, (-i)\, g_0\, e^{-i(\omega_B-\omega_{XH}-\omega_k)t'}\, e^{\Gamma_H t'}\, c_B(t')$$

$$c_{H k_1 k_2}(t) = -i \int_0^t dt'\, g_0\, e^{-i(\omega_{XH}-\omega_{k_2})t'}\, c_{H k_1}(t')$$

Uns interessiert das Langzeitlimit
$t \to \infty$

$c_B(t) \to 0$, $c_{H k_1}(t) \to 0$

$$c_{H k_1 k_2}(\infty) = -\int_0^\infty dt' \, g_0^2 e^{-\Gamma_H t'} e^{-i(\omega_{XH} - \omega_{k_2})t'} \int_0^{t''} dt'' e^{-i(\omega_B - \omega_{XH} - \omega_{k_1})t'' - \Gamma_V t''}$$

Überprüfe Normierung

$$\int dk_1 \int dk_2 \left\{ |c_{H k_1 k_2}(t)|^2 + |c_{V k_1 k_2}(t)|^2 \right\} = 1$$

$$\int dk_1 \int dk_2 \, |c_{H k_1 k_2}(t)|^2 =$$

$$= \int dk_1 \int dk_2 \int dt' \, g_0^2 e^{-\Gamma t' + i(\omega_{XH} - \omega_{k_2})t'} \int dt'' e^{-\Gamma t'' + i(\omega_B - \omega_{XH} - \omega_{k_1})t''}$$

...

k Integrale führen auf $\delta(t' - t''')$ und $\delta(t'' - t'''')$

$$= g_0^4 \frac{4\pi^2}{c^2} \int_0^\infty dt' \, e^{-2\Gamma t'} \left(\frac{1}{-2\Gamma}\right) \left[e^{-2\Gamma t'} - 1 \right]$$

$$= \frac{g_0^4}{(-2\Gamma)} \frac{4\pi^2}{c^2} \left\{ \frac{1}{4\Gamma} [0 + 1] + \frac{1}{2\Gamma} [0 - 1] \right\}$$

$$= \frac{g_0^4 4\pi^2}{c^2 \Gamma^2} \left(\frac{1}{4} - \frac{1}{8} \right) , \quad \Gamma = \frac{\pi g_0^2}{c}$$

$$= \frac{1}{2}$$

$$|\Psi\rangle_{SS} = \int dk_1 \int dk_2 \, \{ c_{Hk_1k_2}(\infty) \, |G, 1_{k_1H}, 1_{k_2H}\rangle + c_{Vk_1k_2}(\infty) \, |G, 1_{k_1V}, 1_{k_2V}\rangle \}$$

Bell-State

Observable ist die $g^{(2)}(t,\tau)$, denn erlaubt die Rekonstruktion der Dichtematrix, oder

Quantum State Tomography

alle Varianten von H und V prinzipiell möglich

$$G^{(2)}_{ijkl}(t,\tau) =$$

$$= {}_{SS}\langle\Psi| E_i^{(-)}(t) \, E_j^{(-)}(t+\tau) \, E_k^{(+)}(t+\tau) \, E_l^{(+)}(t) |\Psi\rangle_{SS}$$

$$l, k, j, i = H, V$$

$$E_{iB}^{(-)}(t) = \int dk \, b_{ki}^{+} \, e^{i\omega_k t} \, \mathcal{E}_k^{i*}$$

$$E_{jX}^{(-)}(t) = \int dk \, a_{kj}^{+} \, e^{i\omega_k t} \, \mathcal{E}_k^{j*}$$

aufgrund unseres Modells wissen wir, dass nur folgende Varianten möglich sind HHHH, VVVV, HHVV, VVHH

$$G^{(2)}_{ijji}(t,\tau) = \left| \, E_{jX}^{(+)}(t+\tau) \, E_{iB}^{(+)}(t) \, |\Psi\rangle \right|^2$$

$$\int dk \int dk' \, \mathcal{E}_k^{i} \, \mathcal{E}_k^{j} \, e^{-i\omega_j(t+\tau)} \, a_{kj} \, e^{-i\omega_i t} \, b_{ki} \, |\Psi\rangle_{SS}$$

sei $v = H$, $i = H$

$$= \int dk \int dk' \, \varepsilon_k^H \varepsilon_k^H \, e^{-i\omega_k'(t+\tau) - i\omega_k t} \, a_{k'H} \, b_{kH}$$

$$\int dk_1 \int dk_2 \; c_{Hk_1k_2} |G, 1_{k_1}, 1_{k_2}\rangle$$

$$[a_{k'H}, a^+_{k_2H}] = \delta(k' - k_2)$$

$$= \int dk \int dk' \int dk_1 \int dk_2 \; \varepsilon_k^2 \, e^{-i \cdots} \int dt' \int dt'' \, a_{k'H} b_{kH} \, a^+_{k_1} b^+_{k_2} |G, vac\rangle$$

$$a_{k'} |vac\rangle = 0$$
$$b_k |vac\rangle = 0$$

$$\varepsilon_k^H \approx \varepsilon_{\bar{k}}^H = const.$$

durch die zwei verbleibenen k Integrale entstehen $\delta(t'' - (t+\tau))$ $\delta(t'' - t)$

$$= -\left(\varepsilon_{\bar{k}}^H\right)^2 \frac{g_0^2 (2\pi)^2}{c^2} e^{-\Gamma_H(t+\tau) - i\omega_{kH}\tau - i\omega_B t}$$

$$E^{(+)}_{HB}(t+\tau) \, E_{HX}(t) \, |\psi\rangle_{SS}$$

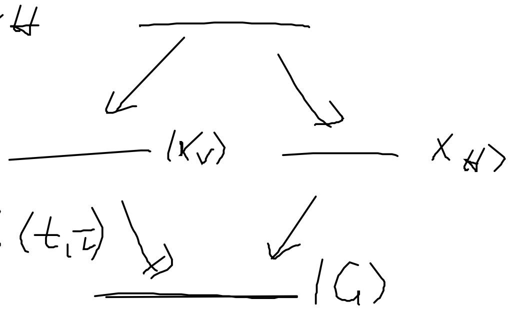
$$= -\left(\varepsilon_{\bar{k}}^H\right)^2 \left(\frac{g_0^2 \, 2\pi}{c}\right)^2 e^{-\Gamma(2t+\tau) - i\omega_{XH}\tau - i\omega_B t}$$

$$G^{(2)}_{HHHH}(t, \tau) = \left(\varepsilon_{\bar{k}}^H \frac{g_0 \, 2\pi}{c}\right)^4 e^{-2\Gamma(2t+\tau)}$$

$$G^{(2)}_{HHVV}(t, \tau) = \left(\sqrt{\varepsilon_{\bar{k}}^H} \sqrt{\varepsilon_{\bar{k}}^V} \, \frac{g_0 \, 2\pi}{c}\right)^4 e^{-2\Gamma(2t+\tau)}$$

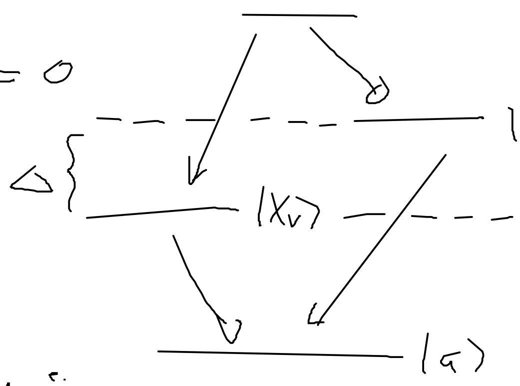
$$e^{-i(\omega_{XH} - \omega_{XV})\tau}$$

falls $\omega_{XV} = \omega_{XH}$



$$G^{(2)}_{HHVV}(t,\tau) =$$
$$= G^{(2)}_{HHHH}(t,\tau) = G^{(2)}_{VVVV}(t,\tau)$$
$$\varepsilon_k^H = \varepsilon_k^V$$

falls $\omega_{XH} - \omega_{XV} \neq 0$
$= \Delta$



Messergebnis ist die aufintegrierte $G^{(2)}$-Funktion

$$\overline{G}^{(2)}_{HHHH} = \int_0^\infty dt \int_0^\infty d\tau \; G^{(2)}_{HHHH}(t,\tau)$$

$$= [\quad]^4 \frac{1}{(-4\Gamma)} [0-1] \frac{1}{(-2\Gamma)} [0-1]$$
$$= \varepsilon^4 \frac{\pi^2}{c^2} 2$$

$$\overline{G}^{(2)}_{HHVV} = \varepsilon^4 \frac{\pi^2}{c^2} 2 \frac{1}{1 - \Delta^2/4\Gamma^2} \longrightarrow 0 \text{ für } \Delta \gg \Gamma$$

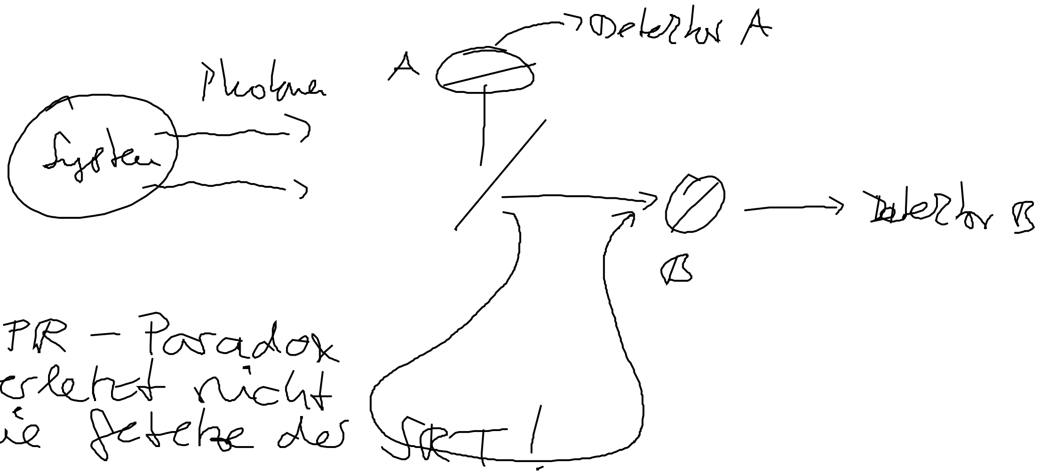
d.h. die Kaskade erzeugt nicht unter allen Umständen ein polarisations-verschränktes Photon-Paar

$$C = 2\left|\frac{\overline{G}^{(2)}_{HHVV}}{\overline{G}^{(2)}_{HHHH}+\overline{G}^{(2)}_{VVVV}}\right| =$$

$$= 2\left|\frac{\varepsilon^4\frac{\pi^2}{c^2}\,2\,\frac{1}{1-i\frac{\Delta}{4n}}}{\varepsilon^4\frac{\pi^2}{c^2}\,2+\varepsilon^4\frac{\pi^2}{c^2}\,2}\right| = \left|\frac{1}{1-i\frac{\Delta}{2n}}\right|$$

$$= \frac{1}{1+\frac{\Delta^2}{4n^2}}$$

$$\overline{\rho} = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{1-i\frac{\Delta}{2n}} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{1+i\frac{\Delta}{2n}} & 0 & 0 & \frac{1}{2} \end{pmatrix}$$



EPR-Paradox verletzt nicht die Gesetze der SRT!

1982: gemessen von Alain Aspect
PRL 49, 1804 (1982)
Loophole frei?