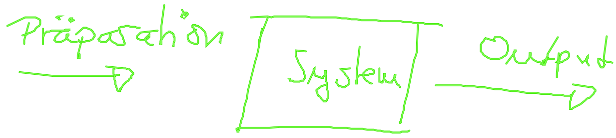
Biexziton-Kaskade II:

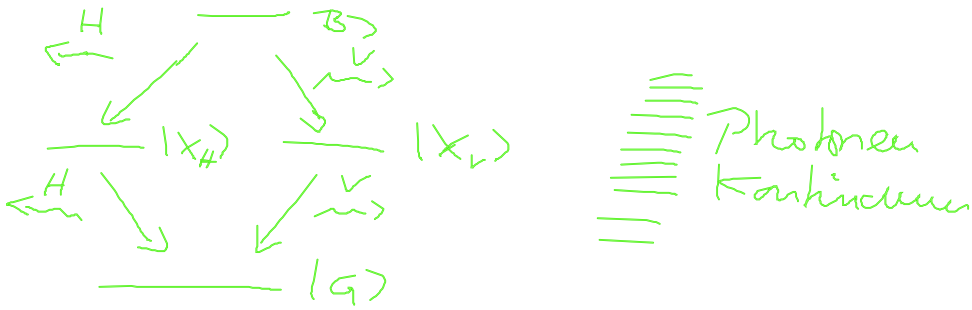
Richard Feynman: „Nobody understands quantum theory." (1967)



$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left\{ |HH\rangle + |VV\rangle \right\}$$

Messbar: niemals die Wellenfunktion, nur die Dichtematrixelemente

$$\rho = \sum_{i,j=H,V} c_{ij} \, |i j\rangle\langle i j|$$



$$H = \int dk \; g_0 \Big\{ e^{i(\omega_B - \omega_{XH} - \omega_k)t} \, b_{kH}^{\dagger} \, B^{+} X_H + h.a.$$

$$+ e^{i(\omega_{XH} - \omega_k)t} \; a_{kH} X_H^{+} G + h.a.$$

$$+ \ldots \{ X_H \text{ in } X_V \text{ ersetzen} \} \Big\}$$

$$|\Psi\rangle = c_B \, |B\rangle + \int dk \{ c_{Hk} \, |X_H, 1_{kH}\rangle + c_{Vk} \, |X_V, 1_{kV}\rangle \}$$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ Biexziton

Ansatz

$$+ \int dk \int dk' \{ c_{Hkk'} \, |G, 1_{kH} \ldots 1_{k'H}\rangle$$

$+ \langle V \aleph k' | G, 1_{kV} \dots 1_{k'V} \rangle$

für den Fall:

$c_B(0) = 1$, ungetriebenes System

Beschränkung auf Zwei-Anregungen

es soll gelten: $a_{kH} | G, 1_{kH} \dots 1_{k'H} \rangle \neq 0$

Schreibweise $\quad a_{kH} | X_H ; 1_{kH} \rangle = c_{kH} b^+_{kH} | X_H, 1 \rangle$

$= 0$

$b_{kH} | X_H, 1_{kH} \rangle \neq 0$

$$i\hbar \partial_t | \psi \rangle = H | \psi \rangle$$

$$g^k_{BH} = g_0 \, e^{i(\omega_B - \omega_{XH} - \omega_k)t}$$

$$g^k_{XH} = g_0 \, e^{i(\omega_{XH} - \omega_k)t}$$

(dasselbe für V)

$$i\hbar \langle B | \dot{\psi} \rangle = \langle B | H | \psi \rangle = \int dk \, \{ g^k_{BH} c_{kH} + g^k_{BV} c_{kV} \}$$

$$\langle X_H, 1_{k_1 H} | \dot{\psi} \rangle = -i \, g^{k_1}_{BH} c_B - i \int dk \, g^k_{XH} c_{Hkk_1}$$

$$\langle G, 1_{k_1 H} 1_{k_2 H} | \dot{\psi} \rangle = \dot{c}_{Hk_1 k_2} = -i \, g^{k_2 *}_{XH} c_{Hk_1}$$

(dasselbe alles für V)

Formal integrieren $\quad c_{Hk_1 k_2}(0) = 0 = c_{Hk_1}(0)$

$$c_{Hk_1 k_2}(t) = -i \int_0^t dt' \, g^{k_2 *}_{XH}(t') \, c_{Hk_1}(t')$$

$$\dot{c}_{H k_1} = -i\, g_{BH}^{k_1 *}(t)\, c_B(t) - + (-i)^2 \int dk\, g_{XH}^{k}(t) \int_0^t dt'\, g_{XH}^{k*}(t')\, c_{Hk_1}(t')$$

$$= -i\, g_{BH}^{k_1 *}(t)\, c_B(t) - \int_0^t dt'\, c_{Hk_1}(t')\, g_0^2 \underbrace{\int dk\, e^{-i(\omega_{XH}-\omega_k)(t-t')}}_{=\frac{2\pi}{c}\delta(t-t')\, e^{-i\omega_{XH}(t-t')}}$$

$$\dot{c}_{Hk_1} = -\Gamma_H\, c_{Hk_1}(t) - i\, g_{BH}^{k_1 *}(t)\, c_B(t)$$

$$c_{Hk_1}(t) = e^{-\Gamma_H t}\int_0^t dt'\,(-i)\, g_{BH}^{k_1 *}(t')\, c_B(t')$$

einsetzen DGL $c_B(t)$

$$\dot{c}_B = -\int dk\, g_{BH}^{k}(t)\, e^{-\Gamma t}\int_0^t dt'\, g_{BH}^{k*}(t')\, c_B(t')\, e^{\Gamma t'}$$

$$+ \{\text{dasselbe in } V\}$$

$$= -\int_0^t dt\, g_0^2\, e^{-\Gamma(t-t')}\, c_B(t') \int dk\, e^{i(\omega_B-\omega_{XH}-\omega_k)(t-t')}$$

$$= -\Gamma_H\, c_B(t) - \Gamma_V\, c_B(t)$$

$$c_B(t) = e^{-(\Gamma_H+\Gamma_V)t}\underbrace{c_B(0)}_{=1}$$

$$c_{Hk_1}(t) = e^{-\Gamma_H t}\int_0^t dt'\,(-i)\, g_0\, e^{-i(\omega_B-\omega_{XH}-\omega_k)t' - \Gamma_H t'}\, c_B(t')$$

$$c_{Hk_1k_2}(t) = -i\int_0^t dt'\, g_0\, e^{-i(\omega_{XH}-\omega_{k_2})t'}\, c_{Hk_1}(t')$$

Uns interessiert der Langzeitlimit

$t \longrightarrow \infty$

$c_B(t) \longrightarrow 0\,,\quad c_{H k_1}(t) \longrightarrow 0$

$$c_{H k_1 k_2}(\infty) = -\int_0^\infty dt'\, g_0^2\, e^{-\Gamma_H t'}\, e^{-i(\omega_{XH}-\omega_{k_2})t'} \int_0^{t'} dt''\, e^{-i(\omega_B-\omega_{XH}-\omega_{k_1})t''-\Gamma_v t''}$$

Überprüfe Normierung

$$\int dk_1 \int dk_2 \left\{ |c_{H k_1 k_2}(t)|^2 + |c_{V k_1 k_2}(t)|^2 \right\} = 1$$

$$\int dk_1 \int dk_2\, |c_{H k_1 k_2}(t)|^2 =$$

$$= \int dk_1 \int dk_2 \int dt'\, g_0^2\, e^{-\Gamma t' + i(\omega_{XH}-\omega_{k_2})t'} \int dt''\, e^{-\Gamma t'' + i(\omega_B-\omega_{XH}-\omega_{k_1})t''}$$

...

k Integrale führen auf $\delta(t'-t''')$ und $\delta(t''-t'''')$

$$= g_0^4 \frac{4\pi^2}{c^2} \int_0^\infty dt'\, e^{-2\Gamma t'} \left(\frac{1}{-2\Gamma}\right)\left[e^{-2\Gamma t'} - 1\right]$$

$$= \frac{g_0^4}{(-2\Gamma)} \frac{4\pi^2}{c^2} \left\{ \frac{1}{4\Gamma}[0+1] + \frac{1}{2\Gamma}[0-1] \right\}$$

$$= \frac{g_0^4\, 4\pi^2}{c^2\Gamma^2} \left(\frac{1}{4} - \frac{1}{8}\right)\,,\quad \Gamma = \frac{\pi g_0^2}{c}$$

$$= \frac{1}{2}$$

$$|\psi\rangle_{SS} = \int d\lambda_1 \int d\lambda_2 \{ c_{H\lambda_1 \lambda_2}(\infty) \, |G, 1_{\lambda_1 H}, 1_{\lambda_2 H}\rangle + c_{V\lambda_1 \lambda_2}(\infty) \, |G, 1_{\lambda_1 V}, 1_{\lambda_2 V}\rangle \}$$

Bell-State

Observable ist die $g^{(2)}(t,\tau)$, denn erlaubt die Rekonstruktion der Dichtematrix", oder

Quantum State Tomography

alle Varianten von H und V prinzipiell möglich

$$G^{(2)}_{ijkl}(t,\tau) =$$

$$= {}_{SS}\langle\psi| E_i^{(-)}(t) \, E_j^{(-)}(t+\tau) \, E_k^{(+)}(t+\tau) \, E_l^{(+)}(t) |\psi\rangle_{SS}$$

$l, k, j, i = H, V$

$$E_{iB}^{(-)}(t) = \int dk \, b^{+}_{ki} \, e^{i\omega_k t} \, \epsilon_k^{i*}$$

$$E_{jX}^{(-)}(t) = \int dk \, a^{+}_{kj} \, e^{i\omega_k t} \, \epsilon_k^{j*}$$

aufgrund unseres Modells wissen wir, dass nur folgende Varianten möglich sind HHHH, VVVV, HHVV, VVHH

$$G^{(2)}_{ijji}(t,\tau) = | E^{(+)}_{jX}(t+\tau) \, E^{(+)}_{iB}(t) |\psi\rangle |^2$$

$$\int dk \int dk' \, \epsilon_k^i \, \epsilon_{k'}^j \, e^{-i\omega_j (t+\tau)} \, a_{kj} \, e^{-i\omega_i t} \, b_{ki} \, |\psi\rangle_{SS}$$

sei $\nu = H$, $i = H$

$$= \int dq \int dq' \, \varepsilon_q^H \varepsilon_q^H \, e^{-i\omega_{q'}(t+\tau) - i\omega_q t} \, a_{q'H} b_{qH}$$

$$\int dq_1 \int dq_2 \; c_{Hq_1q_2} |G, 1_{q_1}, 1_{q_2}\rangle$$

$$[a_{q'H}, a^+_{q_1 H}] = \delta(q' - q_1)$$

$$= \int dq \int dq' \int dq_1 \int dq_2 \; \varepsilon_q^2 \, e^{-i \ldots} \int dt' \int dt'' \, a_{q'H} b_{qH} \, a^+_{q_1} b^+_{q_2} |G, vac\rangle$$

$$a_{q'} |vac\rangle = 0$$

$$b_q |vac\rangle = 0$$

$$\varepsilon_q^H \simeq \varepsilon_{\bar{q}}^H = \text{const.}$$

durch die zwei verbleibenden q Integrale entstehen $\delta(t' - (t+\tau))$ $\delta(t'' - t)$

$$= -\left(\varepsilon_{\bar{q}}^H\right)^2 \frac{g_0^2 (2\pi)^2}{c^2} \, e^{-\Gamma_H(t+\tau) - i\omega_{XH}\tau - i\omega_B t}$$

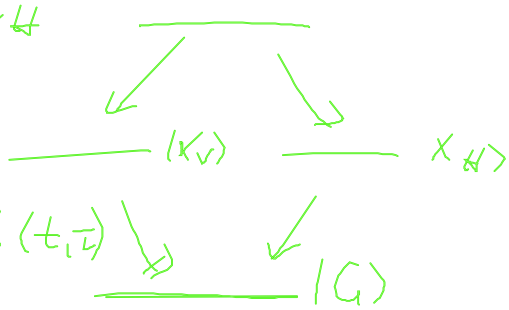
$$E^{(+)}_{HB}(t+\tau) \, E_{HX}(t) \, |\psi\rangle_{SS}$$

$$= -\left(\varepsilon_{\bar{q}}^H\right)^2 \left(\frac{g_0^2 \, 2\pi}{c}\right)^2 e^{-\Gamma(2t+\tau) - i\omega_{XH}\tau - i\omega_B t}$$

$$G^{(2)}_{HHHH}(t,\tau) = \left(\varepsilon_{\bar{q}}^H \frac{g_0 2\pi}{c}\right)^4 e^{-2\Gamma(2t+\tau)}$$

$$G^{(2)}_{HHVV}(t,\tau) = \left(\sqrt{\varepsilon_{\bar{q}}^H} \sqrt{\varepsilon_{\bar{q}}^V} \, \frac{g_0 2\pi}{c}\right)^4 e^{-2\Gamma(2t+\tau)} \, e^{-i(\omega_{XH} - \omega_{XV})\tau}$$

falls $\omega_{XV} = \omega_{XH}$



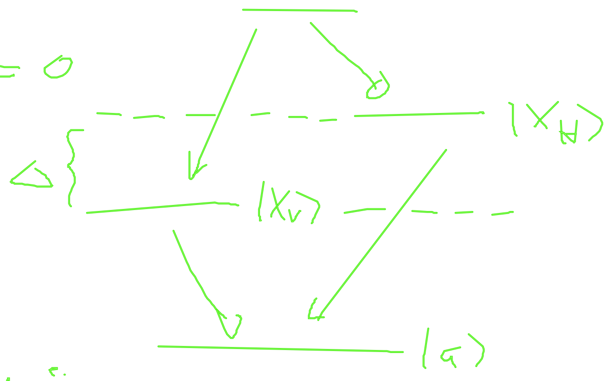
$$G^{(2)}_{HHVV}(t,\tau) =$$

$$= G^{(2)}_{HHHH}(t,\tau) = G^{(2)}_{VVVV}(t,\tau)$$

$$\mathcal{E}^H_k = \mathcal{E}^V_k$$

falls $\omega_{XH} - \omega_{XV} \neq 0$

$= \Delta$



Messergebnis ist die aufintegrierte $G^{(2)}$-Funktion

$$\bar{G}^{(2)}_{HHHH} = \int_0^\infty dt \int_0^\infty d\tau \; G^{(2)}_{HHHH}(t,\tau)$$

$$= [\quad]^4 \frac{1}{(-4\Gamma)} [0-1] \frac{1}{(-2\Gamma)} [0-1]$$

$$= \mathcal{E}^4 \frac{\pi^2}{c^2} 2$$

$$\bar{G}^{(2)}_{HHVV} = \mathcal{E}^4 \frac{\pi^2}{c^2} 2 \frac{1}{1 - \frac{\Delta^2}{4\Gamma^2}} \longrightarrow 0 \text{ für } \Delta \gg \Gamma$$

d.h. die Kaskade erzeugt nicht unter allen Umständen ein polarisations-verschränktes Photon-Paar

$$C = 2\left|\frac{\bar{G}^{(2)}_{HHVV}}{\bar{G}^{(2)}_{HHHH} + \bar{G}^{(2)}_{VVVV}}\right| =$$

$$= 2\left|\frac{\varepsilon^4 \frac{\pi^2}{\Gamma^2}\, 2\, \frac{1}{1 - i\frac{\Delta}{4\Gamma}}}{\varepsilon^4 \frac{\pi^2}{\Gamma^2}\, 2 + \varepsilon^4 \frac{\pi^2}{\Gamma^2}\, 2}\right| = \left|\frac{1}{1 - i\frac{\Delta}{2\Gamma}}\right|$$

$$= \frac{1}{1 + \frac{\Delta^2}{4\Gamma^2}}$$

$$\bar{\rho} = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{1 - \frac{\Delta}{2\Gamma} i} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{1 + \frac{\Delta}{2\Gamma} i} & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

EPR-Paradox verletzt nicht die Gesetze der SRT!

1982: gemessen von Alain Aspect
PRL 49, 1804 (1982)
Loophole frei?