Wdh

feedback control

a.) instantane Messung, stückweise konst. Kontrolle

$\Delta t \to 0 \qquad \dot{\tilde\rho} = \mathcal{L}_{fb}\,\tilde\rho$

$$\boxed{\mathcal{L}_{fb} = \sum_n \mathcal{M}_n \sum_m \mathcal{L}_m \mathcal{M}_m}$$

kond. Liouvillian

$\tilde\rho = \sum_n \mathcal{M}_n \rho$

$\mathcal{M}_m \mathcal{M}_n = \delta_{mn} \cdot \mathcal{M}_n$

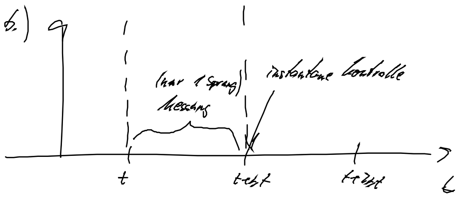
(proj. Messung)

$\mathcal{M}_n \rho \hat{=} |n\rangle\langle n| \rho |n\rangle\langle n|$

$(= M_n \rho M_n^\dagger)$

Beispiel: SET $\to$ eff. Rategleichung $\to$ Verletzung des detl. GG

b.)



$\Delta t \to 0 \qquad \dot\rho = \mathcal{L}_{fb}\,\rho$

$\mathcal{L}_{fb} = -i[H,\rho]$
$+\sum_\alpha \gamma_\alpha \left[U_\alpha L_\alpha \rho L_\alpha^\dagger U_\alpha^\dagger - \frac{1}{2}\{L_\alpha^\dagger L_\alpha, \rho\}\right]$

$\gamma_\alpha \geq 0$

$H_{eff} = H - i\sum_\alpha \frac{\gamma_\alpha}{2} L_\alpha^\dagger L_\alpha$

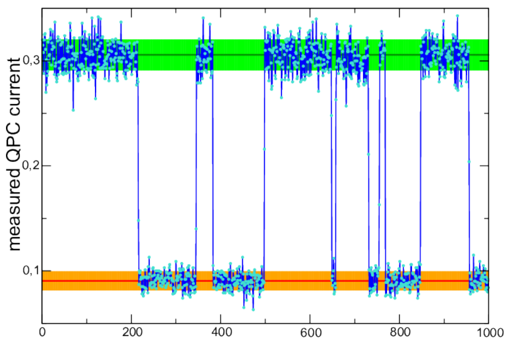
$\rightsquigarrow$ stabilisiere reinen Zustand

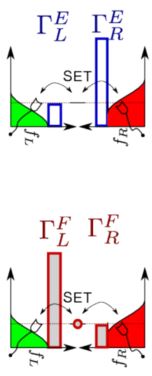
$\mathcal{L}_{fb}(|\psi\rangle\langle\psi|) = 0$

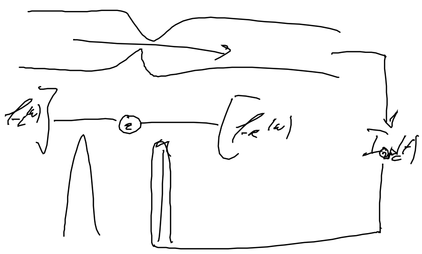
∘ kein Delay zw. Messung & Kontrolle

7. Ausgewählte Anwendungen

7.1. Elektron. Maxwell-Dämon







falls ε = const.

$\Rightarrow$ kein (bzw. vernachlässigbarer) Energietransfer

$$\mathcal{L}_{eff}(\chi_L,\chi_R) = \begin{pmatrix} -\Gamma_L^E f_L - \Gamma_R^E f_R & \Gamma_L^F(1-f_L)e^{+i\chi_L} + \Gamma_R^F(1-f_R)e^{+i\chi_R} \\ +\Gamma_L^E f_L e^{-i\chi_L} + \Gamma_R^E f_R e^{-i\chi_R} & -\Gamma_L^F(1-f_L) - \Gamma_R^F(1-f_R) \end{pmatrix}$$

∘ Trajektorie a.) $P_E(t) = 1 \qquad P_F(t) = 0$

$\rightsquigarrow P_{L,rein} \approx \Gamma_L^E \cdot f_L \cdot \Delta t$

$P_{R,rein} \approx \Gamma_R^E \cdot f_R \cdot \Delta t$

$P_{nichts} = 1 - P_{L,rein} - P_{R,rein}$

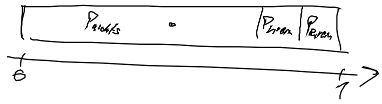
3 Möglichkeiten

b.) $P_E(t) = 0 \qquad P_F(t) = 1$

$P_{L,raus} \approx \Gamma_L^F(1-f_L)\cdot\Delta t$

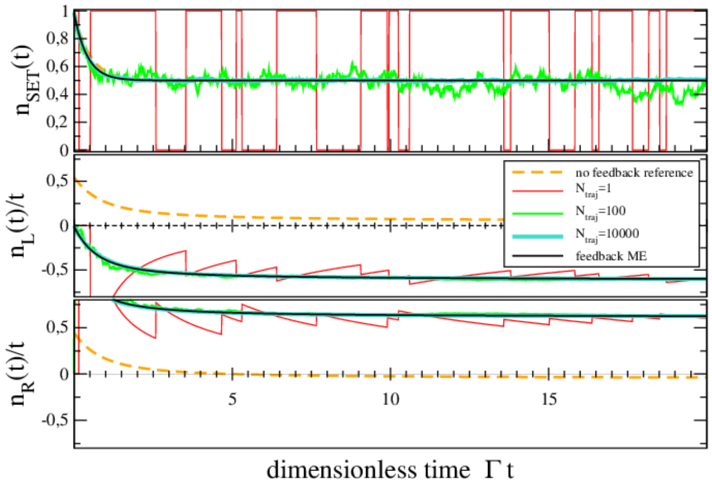
$P_{R,raus} \approx \Gamma_R^F(1-f_R)\cdot\Delta t$

$P_{nichts} = 1 - P_{L,raus} - P_{R,raus}$



$I(f_L - f_R) \neq 0$

$P = -I_n \cdot V = -I_n \cdot (\mu_L - \mu_R)$



$\Gamma_L^E = e^{+\delta}\Gamma \qquad \Gamma_R^E = e^{-\delta}\Gamma \qquad \Gamma_L^F = e^{-\delta}\Gamma \qquad \Gamma_R^F = e^{+\delta}\cdot\Gamma$

$\delta \hat{=}$ feedback-Parameter

$\rightsquigarrow \delta \gg 1 \qquad I_n = e^{\delta}\cdot\Gamma \dfrac{f_L(1-f_R)}{f_L + (1-f_R)} \qquad \beta_L \; \beta_R \quad \mu_L \; \mu_R \qquad P$

falls $\beta_L = \beta_R = \beta$

$\beta(\mu_L - \mu_R) = \ln\left\{\dfrac{f_L(1-f_R)}{(1-f_L)\cdot f_R}\right\} \qquad f_\nu \in [0,1]$

$P \leq k_B T\, \Gamma \cdot e^{\delta} \cdot 0.279 \qquad \Gamma \cdot e^{\delta} \cdot \Delta t < 1$

$W_{dem} \leq k_B T \; 0.279$

$Q \geq k_B T \cdot \ln 2 \approx k_B T \cdot 0.693$

$\Big\}$ keine Verletzung des 2. HS

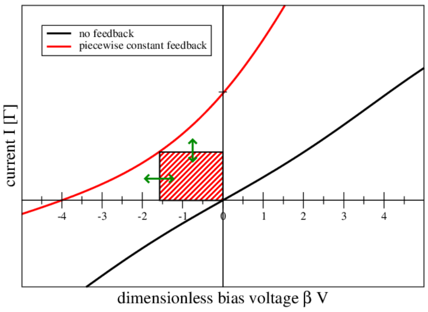
Landauer-Prinzip

CGF: $C(z,t) = \ln M(z,t) = \ln \mathrm{Tr}\{\rho(z,t)\} = \ln \mathrm{Tr}\{e^{\mathcal{L}(z)\cdot t}\rho_0\}$

$\xrightarrow{t\to\infty} \lambda_{dom}(z)\cdot t$

falls $\lambda(-z) = \lambda(+z + i\alpha) \longrightarrow \dfrac{P_n}{P_{-n}} = e^{n\cdot\alpha}$

hier: $\lim\limits_{t\to\infty} \dfrac{P_{+n}(t)}{P_{-n}(t)} = e^{n(\beta\cdot V + 4\delta)} = e^{n\beta(V - V^*)} \qquad V^* = -\dfrac{4\delta}{\beta}$



7.1.2 konventionelle Entropiebilanz in Ratengl.

kein feedback

$$\boxed{\dot P_a = \sum_\nu \sum_b W_{ab}^{(\nu)} P_b}$$

Reservoire

WS für Zustand b

für $a \neq b$ Übergangsrate von $b \to a$ durch Res. ν $\qquad W_{ab}^{(\nu)}\Big|_{a\neq b} \geq 0$

- $W_{aa}^{(\nu)} = -\sum_{b\neq a} W_{ba}^{(\nu)}$ WS-Erhaltung
- Reservoire im GG

Energie von Zust. j $\qquad$ TZ von Zust. j

$$\frac{W_{ji}^{(\nu)}}{W_{ij}^{(\nu)}} = e^{-\beta_\nu[E_j - E_i - \mu_\nu(N_j - N_i)]}$$

$\to$ Entropie $\qquad S(t) = -\sum_i P_i(t) \ln P_i(t)$

$$\dot S(t) = -\sum_i \left[\dot P_i \ln P_i + \cancel{P_i P_i^{-1}} \cdot \dot P_i\right] = -\sum_i \dot P_i \ln P_i$$

$$= -\sum_{ij}\sum_\nu W_{ij}^{(\nu)} P_j \ln\left(P_i \frac{W_{ji}^{(\nu)}}{P_j \cdot W_{ij}^{(\nu)}} \frac{P_j \cdot W_{ij}^{(\nu)}}{W_{ji}^{(\nu)}}\right)$$

$$= \underbrace{+\sum_{ij} W_{ij}^{(\nu)} P_j \ln\left(\frac{W_{ij}^{(\nu)} \cdot P_j}{W_{ji}^{(\nu)} \cdot P_i}\right)}_{\geq 0} + \sum_{ij}\sum_\nu W_{ij}^{(\nu)} P_j \left[\ln\left(\frac{W_{ji}^{(\nu)}}{\cancel{P_j} \cdot W_{ij}^{(\nu)}}\right) - \underbrace{\cancel{\ln P_j}}_{0}\right]$$

~~1~~ Entropie fluss

$$\boxed{\dot S_e^{(\nu)} = \sum_{ij} W_{ij}^{(\nu)} P_j \cdot \ln\frac{W_{ji}^{(\nu)}}{W_{ij}^{(\nu)}} = \beta_\nu\left[I_E^{(\nu)} - \mu_\nu I_N^{(\nu)}\right]}$$

$\to$ log. Summen-Ungl.: $\displaystyle\sum_{i=1}^{n} a_i \cdot \ln\left(\frac{a_i}{b_i}\right) \geq a \cdot \ln\frac{a}{b}$ $\qquad a = \sum_i a_i$, $\quad b = \sum_i b_i$

$a_i, b_i \geq 0$

$$\boxed{\dot S_i = \sum_{ij}\sum_\nu W_{ij}^{(\nu)} P_j \cdot \ln\left[\frac{W_{ij}^{(\nu)} \cdot P_j}{W_{ji}^{(\nu)} \cdot P_i}\right]}$$

"irreversible" Entropie-Prod.-rate

$$\dot S - \underbrace{\sum_\nu \beta_\nu \left[I_E^{(\nu)} - \mu_\nu I_M^{(\nu)}\right]}_{} = \dot S_i \geq 0$$

System-Entropie

Änderung der Entropie in den Bädern

$$dU = T\,dS - \underbrace{p\,dV}_{\to 0} + \mu\,dN \quad \text{in GC}$$

$$dS_{res} = \frac{1}{T}dU_{res} - \frac{1}{T}\mu\,dN_{res} \qquad \frac{dU_{res}}{dt} = -I_E \qquad \frac{dN_{res}}{dt} = -I_M$$

für BMS-Mastergleichung:

$H_S|i\rangle = E_i|i\rangle \qquad [H_S, N_S] = 0$

$N_S|i\rangle = N_i|i\rangle$

$$\langle i|\mathcal{L}^{(\nu)}\rho|i\rangle = \sum_j W_{ij}^{(\nu)}\langle j|\rho|j\rangle$$

$$\dot S_i = -\sum_\nu \mathrm{Tr}\left\{\mathcal{L}^{(\nu)}\rho\left[\ln\rho - \ln\bar\rho^{(\nu)}\right]\right\}$$

$$= \sum_\nu \left[\underbrace{\dot S_{i,1}^{(\nu)}}_{\geq 0} + \underbrace{\dot S_{i,2}^{(\nu)}}_{\geq 0}\right]$$

entspricht der Ratengleichung

7.1.3. Entropiebilanz mit feedback

feedback

$$\dot P_i = \sum_\nu \sum_j W_{ij}^{(j,\nu)} P_j$$

Energie-Übertrag bei $j \to i$:

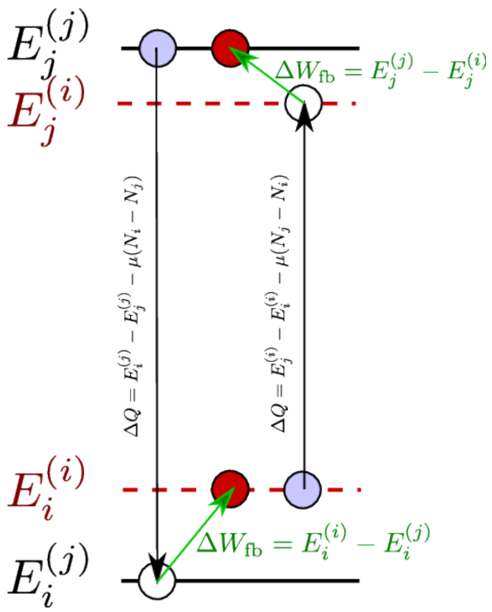
$$\Delta E_{ij} = \underbrace{\left(E_i^{(j)} - E_j^{(j)}\right)}_{\hookrightarrow \text{Wärme}} + \underbrace{\left(E_i^{(i)} - E_i^{(j)}\right)}_{\text{Kontroll-Arbeit}}$$

$$I_E^{(\nu)} = \sum_{ij}\left[E_i^{(j)} - E_j^{(j)}\right] W_{ij}^{(j\nu)} \cdot P_j$$

$$I_M^{(\nu)} = \sum_{ij}(N_i - N_j) \cdot W_{ij}^{(j\nu)} \cdot P_j$$

$$I_E^{fb} = \sum_\nu \sum_{ij}\left(E_i^{(i)} - E_i^{(j)}\right) W_{ij}^{(j\nu)} \cdot P_j$$

$$\dot E = \underbrace{\sum_\nu \mu_\nu \cdot I_M^{(\nu)}}_{\text{chem. Arbeit}} + \underbrace{I_E^{fb}}_{\text{Kontrollarbeit}} + \underbrace{\sum_\nu \left[I_E^{(\nu)} - \mu_\nu I_M^{(\nu)}\right]}_{\text{Wärme}}$$



$$\dot S = \dot S_i + \dot S_e$$

$$\sum_\nu \sum_{ij} W_{ij}^{(j\nu)} P_j \cdot \ln\left[\frac{W_{ij}^{(j\nu)} P_j}{W_{ji}^{(i\nu)} \cdot P_i}\right] \geq 0$$

$$\frac{W_{ij}^{(j\nu)}}{W_{ji}^{(i\nu)}} = e^{\beta_\nu\left[\left[E_i^{(j)} - E_j^{(j)}\right] - \mu_\nu(N_i - N_j)\right]} e^{-\epsilon_{ij}^{(\nu)}} e^{-\sigma_{ij}^{(\nu)}}$$

im Endeffekt:

$$\dot S_i = \dot S - \sum_\nu \beta_\nu \dot Q^{(\nu)} + I_1 + I_2 \geq 0$$

$$I_2 = \sum_\nu \sum_{ij} W_{ij}^{(j\nu)} P_j \cdot \sigma_{ij}^{(\nu)}$$

$$I_1 = \sum_\nu \sum_{ij} W_{ij}^{(j\nu)} P_j \cdot \epsilon_{ij}^{(\nu)}$$

$\to$ Entropie (bzw. Informationsströme) durch die Kontrollschleife müssen berücksichtigt werden.