

Wdh

o Floquet:  $\rho(t) = \text{Tr}(\rho(t)) \cdot e^{-iH \cdot t} = \sum_n e^{-iE_n \cdot t} |\psi(t)\rangle \langle \psi(0)|$

$\rho(t) = \sum_n e^{+iE_n(t)} \cdot \exp\left\{-i \int_0^t E_n(t') dt'\right\} |\psi(t)\rangle \langle \psi(0)|$

Berry-Phase  $H(t)|\psi(t)\rangle = E_n(t)|\psi(t)\rangle$

o POVM  $\{M_n\}$  Messoperatoren :  $\sum_n M_n^\dagger M_n = \mathbb{1}$   
 $n \hat{=} \text{Messergebnis}$

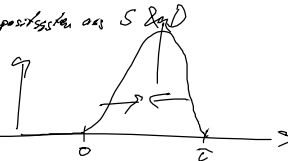
$\rho \rightarrow \rho^{(n)} = \frac{M_n \rho M_n^\dagger}{\text{Tr}\{M_n^\dagger M_n \rho\}} = \frac{M_n \rho M_n^\dagger}{p(n)}$

- + falls  $M_n = |a\rangle\langle a| \rightarrow$  projektive Messung
- + alle Dir-Eigenschaften bleiben erhalten
- + ergibt sich aus projektiver Messung auf Kompositensystem aus  $S$  und  $D$

+ Bsp:  $H(t) = H_0 + H_D + g(t)H_{SD}$

$\rho(t) = \hat{U} \exp\left\{-i \int_0^t H(t') dt'\right\} \rho(0)$

$\Rightarrow |t \rightarrow 0\rangle \quad \rho_0 = e^{-i \int_0^t H_{SD} dt}$



$z = \int_0^\tau g(t) dt$

$p(0) = \langle \uparrow | \rho_0^\dagger | \uparrow \rangle + \langle \downarrow | \rho_0^\dagger | \downarrow \rangle \cdot e^{-2z}$

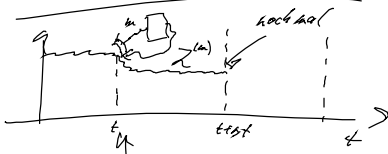
$p(n \geq 1) = \frac{z^{2n} e^{-2z}}{n!} \langle \downarrow | \rho_0^\dagger | \downarrow \rangle \quad \sum_{n=1}^\infty p(n) = (1 - e^{-2z}) \langle \downarrow | \rho_0^\dagger | \downarrow \rangle$

falls  $\rho_0^\dagger = |\uparrow\rangle\langle\uparrow| \quad |z\rangle = \frac{z}{\sqrt{2}} (|\uparrow\rangle + |\downarrow\rangle)$

## 6.2. Externe stochastische konstante Kontrollfelder

- kein Delay zw. Messung & Kontrolle
- Kontrolloperatoren seien mundtot schnell

### 6.2.1. Wiederholte FB-Operationen



Messung am Anfang des Intervalls

$\rho^{(n)} = \frac{M_n \rho M_n^\dagger}{p(n)} = \frac{1}{p(n)} \text{Tr}\{M_n^\dagger M_n \rho\}$

$\rho^{(n)}(t+\delta t) = e^{Z^{(n)} \cdot \delta t} M_n \rho(t) \cdot \frac{1}{p(n)}$

$\rho(t+\delta t) = \sum_n p(n) \rho^{(n)}(t+\delta t) = \sum_n e^{Z^{(n)} \cdot \delta t} M_n \rho(t) = \rho(t+\delta t)$

(noch allgemeiner)  
 $\rho(t+\delta t) = \sum_n K^{(n)}(\delta t, t) M_n \rho(t)$

6.2.2. Kohärentes Feedback ( $t \rightarrow \infty$ )

$$\rho(t+\delta t) = \left[ \sum_n M_n + \delta t \sum_n Z^{(n)} M_n \right] \rho(t) + \mathcal{O}(\delta t^2) \quad \left( \sum_n M_n \right)$$

wissen:  $\sum_n M_n^\dagger M_n = \mathbb{1}$  aber  $\sum_n M_n \neq \mathbb{1}$       $\sum_n M_n \rho = \sum_n M_n \rho M_n^\dagger \neq \rho$

$M_0 = 19 \times 91$       $M_1 = 16 \times 61$

$$\rho = \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix} \rightarrow M_0 \rho M_0^\dagger + M_1 \rho M_1^\dagger = \begin{pmatrix} \rho_{00} & 0 \\ 0 & \rho_{11} \end{pmatrix}$$

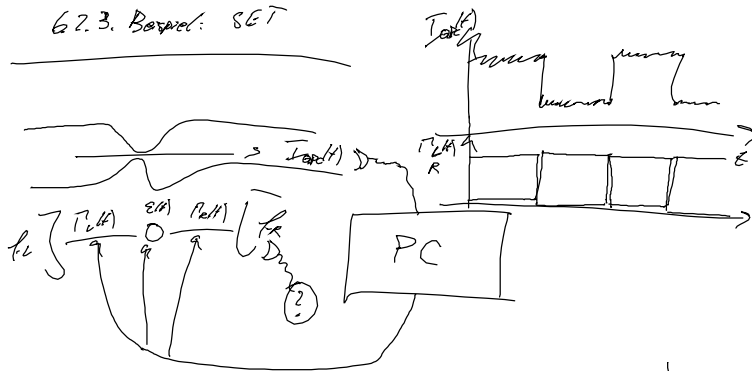
für projektive Messungen:  $M_n^\dagger M_n = M_n \cdot \delta_{nn}$

$$\underbrace{\sum_n M_n \rho(t+\delta t)}_{\tilde{\rho}(t+\delta t)} - \underbrace{\sum_n M_n \sum_n M_n \rho(t)}_{\tilde{\rho}(t)} = \delta t \sum_n M_n \sum_n Z^{(n)} M_n \underbrace{\sum_n M_n \rho(t)}_{\tilde{\rho}(t)}$$

für  $t \rightarrow \infty$

$$\tilde{\rho} = \sum_{\text{FB}} \tilde{\rho} \quad \sum_{\text{FB}} = \sum_n M_n \sum_n Z^{(n)} M_n$$

6.2.3. Beispiel: SET



$$\text{vec}(M_0 \rho M_0^\dagger) = M_0 \text{vec}(\rho) \quad \text{vec}(\rho) = \begin{pmatrix} \rho_{00} \\ \rho_{01} \\ \rho_{10} \\ \rho_{11} \end{pmatrix}$$

$$\rho = \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix} \quad M_0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \rightarrow M_0 \rho M_0^\dagger = \begin{pmatrix} \rho_{00} & 0 \\ 0 & 0 \end{pmatrix} \Rightarrow M_0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$Z^{(0)} = \begin{pmatrix} Z_{PP}^{(0)} & \textcircled{1} \\ \textcircled{1} & Z_{RR}^{(0)} \end{pmatrix} \quad Z^{(1)} = \begin{pmatrix} Z_{PP}^{(1)} & \textcircled{1} \\ \textcircled{1} & Z_{RR}^{(1)} \end{pmatrix}$$

$$M_1 = \begin{pmatrix} 0 & & \textcircled{1} \\ & 1 & \\ \textcircled{1} & & 0 \\ & & & 0 \end{pmatrix}$$

$M_0 M_1 \neq \mathbb{1}_4$

$$\lambda_{H\pm} = \left( \begin{array}{cc|cc} -\Gamma_L^0 \rho_L^0 - \Gamma_R^0 \rho_R^0 & +\Gamma_L^0 (1-\rho_L^0) + \Gamma_R^0 (1-\rho_R^0) & 0 & 0 \\ +\Gamma_L^0 \rho_L^0 + \Gamma_R^0 \rho_R^0 & -\Gamma_L^0 (1-\rho_L^0) - \Gamma_R^0 (1-\rho_R^0) & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) \quad M_0 + M_1 = \left( \begin{array}{c|c} \text{4} & \text{4} \\ \hline \text{4} & \text{4} \end{array} \right)$$

lokales det. GG wird verletzt  
→ schlechte Verletzung des 2. HS

$$= \left( \begin{array}{c|c} \lambda_{H\pm}^{\text{prop}} & \text{4} \\ \hline \text{4} & \text{4} \end{array} \right)$$

### 6.3. Wiseman-Milburn-Kontrolle

#### 6.3.1. Dissipative Kontrollschleifen

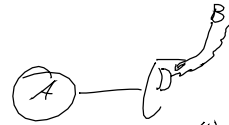
System (A) + Detektor (B)    Operator A    Basis in B

Komposit-DH:  $\mathcal{E}(H) = \sum_{h \in \mathcal{H}} \rho^{(h)} \otimes |h\rangle\langle h|$

FCS  $\rho^{(h)} = \rho^{(h,h)}$

Sprünge

$$\dot{\rho}^{(h)}(t) = \underbrace{\mathcal{L}_0 \rho^{(h)}(t)}_{\text{kon. Sprung}} + \mathcal{L}_+ \rho^{(h-1)}(t) + \mathcal{L}_- \rho^{(h+1)}(t)$$



$$\rho^{(h)}(t) = \frac{1}{\mathcal{Z}(t)} \int_{-\infty}^{+\infty} \rho(z,t) e^{-izt} dz$$

$$\rho(z,t) = \sum_h \rho^{(h)}(t) e^{+izt}$$

$$\mathcal{Z}(z) = \mathcal{Z}_0 + e^{+iz_0} \mathcal{Z}_+ + e^{-iz_0} \mathcal{Z}_-$$

$$\mathcal{E}(H(t)) = \mathcal{E}(H) + \Delta t \sum_{h \in \mathcal{H}} \dot{\rho}^{(h)}(t) \otimes |h\rangle\langle h| + \mathcal{O}(\Delta t^2)$$

$$= \mathcal{E}(H) + \Delta t \sum_{h \in \mathcal{H}} \left[ \mathcal{L}_0 \rho^{(h)}(t) + \mathcal{L}_+ \rho^{(h-1)}(t) + \mathcal{L}_- \rho^{(h+1)}(t) \right] \otimes |h\rangle\langle h| + \Delta t \sum_{h \in \mathcal{H}} \dot{\rho}^{(h)}(t) \otimes |h\rangle\langle h| + \dots$$

fliegt raus wg.  $h \neq h$

jetzt: bei t sei der Detektor in ZS  $|0\rangle\langle 0|$      $\mathcal{E}(H) = \rho(H) \otimes |0\rangle\langle 0|$

$$M_0(\Delta t) \rho(H) = \text{Tr}_B \left\{ |0\rangle\langle 0| \mathcal{E}(H(t+\Delta t)) |0\rangle\langle 0| \right\} = \langle 0| \mathcal{E}(H(t+\Delta t)) |0\rangle = \left( \frac{1}{\mathcal{Z}} + \mathcal{Z}_0 \cdot \Delta t \right) \cdot \rho(H)$$

$$\mathcal{E}(H(t+\Delta t)) = \rho(H) \otimes |0\rangle\langle 0| + \Delta t \left[ \mathcal{L}_0 \rho(H) \otimes |0\rangle\langle 0| + \mathcal{L}_+ \rho(H) \otimes |1\rangle\langle 1| + \mathcal{L}_- \rho(H) \otimes |-1\rangle\langle -1| \right] + \text{off.} + \mathcal{O}(\Delta t^2)$$

$$M_{\pm 1}(\Delta t) \rho(H) = \Delta t \cdot \mathcal{L}_{\pm} \rho(H)$$

$$M_0(\Delta t) + M_{-1}(\Delta t) + M_{+1}(\Delta t) = \frac{1}{\mathcal{Z}} + \mathcal{Z}_0 \cdot \Delta t + \mathcal{L}_{-1} \cdot \Delta t + \mathcal{L}_{+1} \cdot \Delta t$$

$$\mathcal{U}_{\pm} \text{vec}(\rho) = \text{vec}(u_{\pm} \rho u_{\pm}^{\dagger}) \quad u_{\pm} = e^{-iV}$$

$$\rho(t+\delta t) = [M_0(\delta t) + M_+ M_- \delta t + M_- M_+ \delta t] \rho(t)$$

$$= \left\{ 1 + \delta t \underbrace{[L_0 + M_+ L_+ + M_- L_-]}_{Z_H} \right\} \rho(t)$$

$$Z_H = L_0 + M_+ L_+ + M_- L_-$$

$$L = L_0 + L_+ + L_-$$

Spursystem

$$\dot{\rho} = -i [H, \rho] + \sum_{\alpha} \gamma_{\alpha} \left[ L_{\alpha} \rho L_{\alpha}^{\dagger} - \frac{1}{2} \{L_{\alpha}^{\dagger} L_{\alpha}, \rho\} \right] = Z \rho$$

$$Z_H \rho = -i [H, \rho] + \sum_{\alpha} \gamma_{\alpha} \left[ \underbrace{M_{\alpha} L_{\alpha} \rho L_{\alpha}^{\dagger} M_{\alpha}^{\dagger}}_{L_{\alpha}} - \frac{1}{2} \{L_{\alpha}^{\dagger} L_{\alpha}, \rho\} \right]$$

Anwendung: dissipative FB-Präparation von Zuständen durch geeignete Kontrolle der

Umschreibung

$$\dot{\rho} = -i \underbrace{[H - i \sum_{\alpha} \frac{\gamma_{\alpha}}{2} L_{\alpha}^{\dagger} L_{\alpha}]}_{H_{eff}} \rho + i \rho \underbrace{[H + i \sum_{\alpha} \frac{\gamma_{\alpha}}{2} L_{\alpha}^{\dagger} L_{\alpha}]}_{H_{eff}^{\dagger}} + \sum_{\alpha} \gamma_{\alpha} M_{\alpha} L_{\alpha} \rho L_{\alpha}^{\dagger} M_{\alpha}^{\dagger}$$

"nicht-hermitescher H eff."

$\in \mathbb{C}$

$$H_{eff} |\varphi\rangle = \varepsilon |\varphi\rangle$$

wählen  $Z_{eff}(17 \times 71) = 0$

$$\langle \varphi | H_{eff}^{\dagger} = \langle \varphi | \varepsilon^*$$

$$0 = -i \varepsilon (17 \times 71) + i \varepsilon^* (17 \times 71) + \sum_{\alpha} \gamma_{\alpha} M_{\alpha} L_{\alpha} (17 \times 71) L_{\alpha}^{\dagger} M_{\alpha}^{\dagger}$$

$$2 \cdot \text{Im}(\varepsilon) + \sum_{\alpha} \gamma_{\alpha} |b_{\alpha}|^2 = 0$$

$$M_{\alpha} L_{\alpha} |\varphi\rangle = \sum_{\alpha} \gamma_{\alpha} |\varphi\rangle$$

$\uparrow$   
 $\in \mathbb{C}$

ermöglicht Stabilisierung von  $17 \times 71$