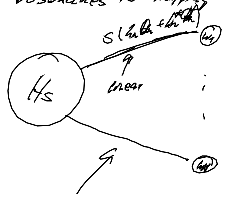
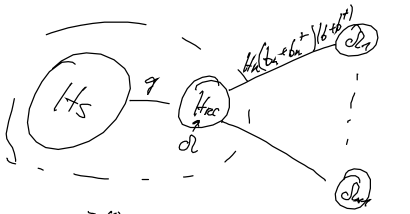
Wdh: • bosonisches RC mapping



RC-mapping $\Longrightarrow$



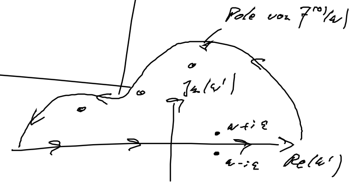
$J^{(0)}(\omega) = 2\pi \sum_k |h_k|^2 \delta(\omega-\omega_k)$

originale SD

$J^{(1)}(\omega) = 2\pi \sum_k |H_k|^2 \delta(\omega-\Omega_k)$

residuale SD

$$\boxed{\Omega_0^2 = \frac{\int_0^\infty \omega\, J^{(0)}(\omega)\, d\omega}{\int_0^\infty \frac{J^{(0)}(\omega)}{\omega}\, d\omega} \qquad q^2 = \frac{1}{2\pi\Omega_0}\int \omega \cdot J^{(0)}(\omega)\cdot d\omega \qquad J^{(1)}(\omega) = \frac{4q^2\, J^{(0)}(\omega)}{\left[\frac{1}{\pi}\mathcal{P}\int\frac{J^{(0)}(\omega')}{\omega-\omega'}d\omega'\right]^2 + \left[J^{(0)}(\omega)\right]^2}}$$



$$\mathcal{P}\int_{-\infty}^{+\infty}\frac{J^{(0)}(\omega')}{\omega-\omega'}d\omega' = \lim_{\varepsilon\to0}\int\frac{J^{(0)}(\omega')(\omega-\omega')}{(\omega-\omega')^2+\varepsilon^2}$$

$[\omega-\omega'-i\varepsilon][\omega-\omega'+i\varepsilon]$

isolierte Pole

$$= 2\pi i \sum_k \underset{\omega'=\omega_k}{\mathrm{Res}}\frac{J^{(0)}(\omega')}{\omega-\omega'} + \underbrace{2\pi i \lim_{\varepsilon\to0}\underset{\omega'=\omega+i\varepsilon}{\mathrm{Res}}\frac{J^{(0)}(\omega')\overbrace{(\omega-\omega')}^{-i\varepsilon}}{\underbrace{[\omega-\omega'-i\varepsilon]}_{\to -2i\varepsilon}[\omega-\omega'+i\varepsilon]}}_{\frac{J^{(0)}(\omega)}{2}}$$

$\Rightarrow$ ME für Supersystem (System + RC + Kopplung)

$\dot\rho = \mathcal{L}\rho \rightsquigarrow \rho(t) = e^{\mathcal{L}t}\rho_0$ Markovsch

$\rho_S(t) = \mathrm{Tr}_{RC}\{\rho(t)\} \Rightarrow \mathrm{Tr}_{RC}\{\mathcal{L}\rho(t)\} = \dot\rho_S$

i.A. nicht-Markovsch

4.4. Fermionische RC Abbildung

4.4.1. Fermionische BTs

$c_k = \sum_q u_{kq} d_q + v_{kq} d_q^\dagger$

$u \qquad v$

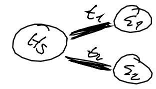
Erhalte Anti-Kommutator Relationen

$$\left.\begin{aligned} uu^\dagger + vv^\dagger &= 1\\ uv^T + vu^T &= 0\end{aligned}\right\} \xrightarrow{v=0} u \text{ muss unitär sein für } v=0$$

hier: $V=0$ (keine Mischung)

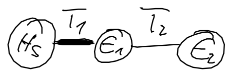
4.4.2. 2 Moden-Beispiel

$$H = H_S + (t_1 d^\dagger c_1 + t_1^* c_1^\dagger d) + (t_2 d^\dagger c_2 + t_2^* c_2^\dagger d) + \varepsilon_1 c_1^\dagger c_1 + \varepsilon_2 c_2^\dagger c_2$$



$$= [\; c_1 = u_{11} d_1 + u_{12} d_2 \quad c_2 = u_{21} d_1 + u_{22} d_2 \;]$$

$$= H_S + [d^\dagger (t_1 \cdot u_{11} + t_2 \cdot u_{21}) d_1 + d^\dagger \underbrace{(t_1 \cdot u_{12} + t_2 u_{22})}_{\stackrel{!}{=}0} d_2 + h.c.]$$
$$+ \varepsilon_1 [\ldots][\ldots]^\dagger + \varepsilon_2 [\ldots][\ldots]$$



$$\boxed{U = \frac{1}{\sqrt{|t_1|^2+|t_2|^2}} \begin{pmatrix} +t_1^* & -t_2 \\ +t_2^* & +t_1 \end{pmatrix}} \to T_1, T_2, \underbrace{E_1 / E_2}$$

$$T_1 = \sqrt{|t_1|^2 + |t_2|^2} \xrightarrow{t_i \to \infty} \infty$$

$$T_2 = \frac{t_1 \cdot t_2}{|t_1|^2 + |t_2|^2} (\varepsilon_2 - \varepsilon_1) \xrightarrow{t_i \to \infty} \text{bleibt endlich}$$

4.4.3. Ableitung der Abbildung

$$H = H_S + c \sum_k t_k^* c_k^\dagger - c^\dagger \sum_k t_k c_k + \sum_k \varepsilon_k c_k^\dagger c_k \quad \to \Gamma^{(0)}(\omega) = 2\pi \sum_k |t_k|^2 \delta(\omega - \varepsilon_k)$$

$$\stackrel{!}{=} H_S + \lambda c d^\dagger - \lambda c^\dagger d + E d^\dagger d + d \sum_k T_k^* d_k^\dagger - d^\dagger \sum_k T_k d_k + \sum_k E_k d_k^\dagger d_k$$

RC

$$\to \Gamma^{(1)}(\omega) = 2\pi \sum_q |T_k|^2 \delta(\omega - E_k)$$

$$c_k = \sum_q u_{kq} d_q \qquad d = d_0$$

- Terme mit $c/c^\dagger \to$ Kopplung

$$\lambda \cdot d = \sum_k t_k \cdot c_k \qquad \lambda d^\dagger = \sum_k t_k^* c_k^\dagger \quad \Rightarrow \{\lambda d, \lambda d^\dagger\} = \lambda^2 = \sum_{kq} t_k t_q^* \underbrace{\{c_k, c_q^\dagger\}}_{\delta_{kq}}$$

$$\Rightarrow \lambda^2 = \sum_k |t_k|^2 = \boxed{\frac{1}{2\pi} \int_{-\infty}^{\infty} \Gamma^{(0)}(\omega)\, d\omega = \lambda^2} \text{ Kopplung System - RC}$$

- Energie

$$d = \sum_k \frac{t_k}{\lambda} c_k = \sum_k (U^\dagger)_{0k} c_k = \sum_k u_{k0}^* c_k \quad \to u_{k0} = \frac{t_k^*}{\lambda}$$

$$E d^\dagger d = \sum_k \varepsilon_k \cdot |u_{k0}|^2 d^\dagger d \quad \Rightarrow \quad E = \sum_k \varepsilon_k \frac{|t_k|^2}{\lambda}$$

$$\Rightarrow \boxed{E = \frac{1}{2\pi \lambda^2} \int_{-\infty}^{\infty} \omega \cdot \Gamma^{(0)}(\omega)\, d\omega}$$

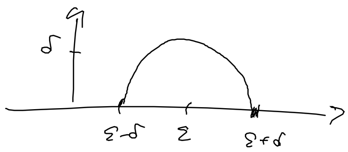
$$\boxed{\Gamma^{(1)}(\omega) = \frac{4\lambda^2 \, \Gamma^{(0)}(\omega)}{\left[\frac{1}{\pi} \mathcal{P}\int_{-\infty}^{\infty} \frac{\Gamma^{(0)}(\omega')}{\omega - \omega'} d\omega'\right]^2 + \left[\Gamma^{(0)}(\omega)\right]^2}}$$

Unterschiede: q vs λ anders
Ω vs E
$\eta(\omega) = -\eta(-\omega)$ vs. $\Gamma(\omega) \geq 0 \;\forall \omega$

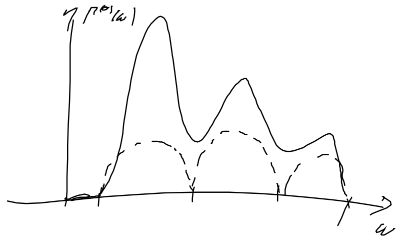
- $\Gamma^{(0)}(\omega) \to \alpha \cdot \Gamma^{(0)}(\omega) \quad \to \Gamma^{(1)}$ ist invariant

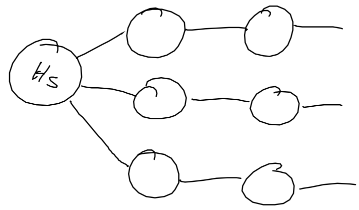
- rekursive Anwendung möglich
- $\Gamma(\omega) = \delta\sqrt{1-\left(\frac{\omega-\varepsilon}{\delta}\right)^2}\cdot\Theta\left(\delta^2-(\omega-\varepsilon)^2\right)$

  nur für $\Gamma^{(0)}(\omega)\neq 0$ $\omega\in[\varepsilon-\delta,\varepsilon+\delta]$

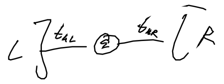


- Anwendung





4.4.4 SET (Beispiel)



$$H = \varepsilon d^\dagger d + \sum_\nu\sum_k\left(\varepsilon_{k\nu} c_{k\nu}^\dagger c_{k\nu} + t_{k\nu} d^\dagger c_{k\nu} + t_{k\nu}^* c_{k\nu}^\dagger d\right)$$

$$\Gamma_\nu^{(0)}(\omega) = \frac{\Gamma_\nu\,\delta_\nu^2}{(\omega-\varepsilon_\nu)^2+\delta_\nu^2}$$

δ_ν^2 ← Breite

Höhe d. Maximums; Position d. Max.

wideband: $\delta_\nu\to\infty$ $\Gamma_\nu^{(0)}(\omega)\Rightarrow\Gamma_\nu$

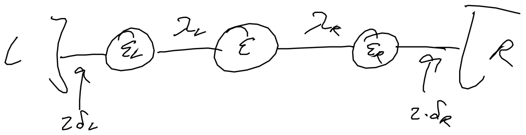
1 RC für jedes Reservoir

linke RC; rechte RC

$$\sqrt{\frac{\Gamma_L\delta_L}{2}}\qquad\sqrt{\frac{\Gamma_R\delta_R}{2}}$$

$$\Rightarrow H = \underbrace{\varepsilon d^\dagger d + \varepsilon_L d_L^\dagger d_L + \varepsilon_R d_R^\dagger d_R + \left(\lambda_L d^\dagger d_L + \lambda_R d^\dagger d_R + h.c.\right)}_{H_S' = H_{TQD}}$$

$$+\sum_\nu\sum_k\left[\epsilon_{k\nu} d_{k\nu}^\dagger d_{k\nu} + \left(T_{k\nu} d_\nu^\dagger d_{k\nu} + h.c.\right)\right]$$



$\Gamma_\nu^{(1)}(\omega) = 2\cdot\delta_\nu$ wird konstant

Vergleiche

i.) exakte Lösung

$I_M = \int d\omega\, T(\omega)\left[f_L(\omega)-f_R(\omega)\right]$ geht zu berechnen

ii.) naive ME für SET

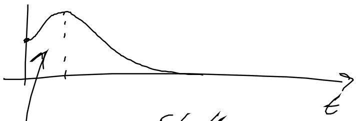
$$I_M = \frac{\Gamma_L\Gamma_R}{\Gamma_L+\Gamma_R}\left[f_L(\varepsilon)-f_R(\varepsilon)\right]\xrightarrow{\Gamma_L=\Gamma_R=\Gamma}\frac{\Gamma}{2}\left[f_L(\varepsilon)-f_R(\varepsilon)\right]$$

iii.) ME für TQD (supersystem): BMS

$\dot\rho = \mathcal{L}_{BMS}\,\rho$ Markovsch (weil $\mathcal{L}_{BMS}$ = Lindblad-Form) $\qquad\frac{d}{dt}D(\rho(t)\,\|\,\bar\rho)\leq 0$

$$\rho_c(t) = \mathrm{Tr}_{LR}\{\rho(t)\} = \begin{pmatrix}1-P_1(t) & 0\\ 0 & P_1(t)\end{pmatrix}\qquad P_1(t) = \mathrm{Tr}\{d^\dagger d\,\rho(t)\}$$

$D(\rho_c(t)\,\|\,\bar\rho_c)$



Markovsche Einbettung

nicht-Markovsche Entwicklung

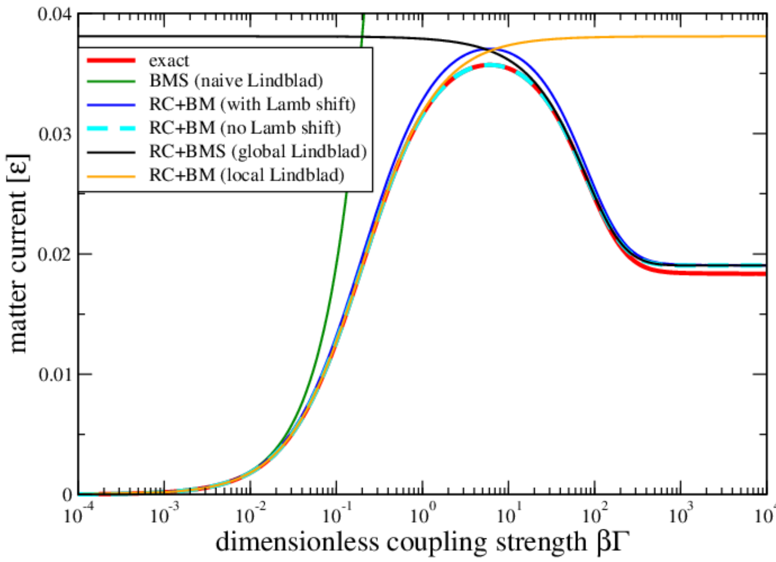
iii.) nicht säkulare -ME für TQD

iv.) lokale ME für Supersystem

$$\dot{\rho} = -i[H_{TQD}, \rho]$$

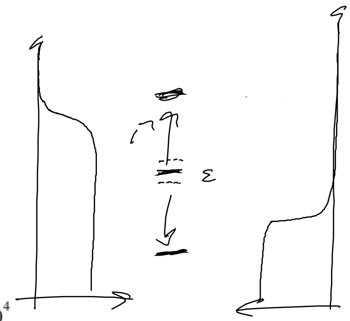
$$+ \Big[+ \Gamma_L^{(1)}(\epsilon_L)[1-f_L(\epsilon_L)]\left[d_L \rho d_L^\dagger - \tfrac{1}{2}\{d_L^\dagger d_L, \rho\}\right] + \Gamma_L^{(1)}(\epsilon_L) f_L(\epsilon_L) \, (d_L \leftrightarrow d_L^\dagger)$$

$$+ (L \to R) \Big]$$



$\mu_L \neq \mu_R$

$\Gamma_L = \Gamma_R = \Gamma$



4.5. Stationärer Zustand

$$\bar{\rho}_S' = \frac{e^{-\beta H_S'}}{Z'} \qquad \text{Gibbs-Zustand des Supersystems}$$

$$\bar{\rho}_S = \mathrm{Tr}_B\left\{\frac{e^{-\beta(H_S+H_B+H_I)}}{Z}\right\} \xrightarrow[Z = Z_S \cdot Z_B]{H_I \to 0} \frac{e^{-\beta H_S}}{Z_S} \qquad \text{(lokaler Gibbs)}$$

globaler Gibbs

Supersystem: $H_S' = H_S + H_{RC} + H_I$

$$e^{-\beta H^*} \equiv \frac{\mathrm{Tr}_B\{e^{-\beta(H_S+H_I+H_B)}\}}{\mathrm{Tr}_B\{e^{-\beta H_B}\}}$$

für $H_I \to 0$: $e^{-\beta H^*} = e^{-\beta H_S}$

Hamiltonian of mean force $\xrightarrow{H_I \to 0} H^* \to H_S$

"effektiver" System-Hamiltonian in Limes starker Kopplung

$$e^{-\beta H^*} = \frac{\mathrm{Tr}_{RC,B'}\left\{e^{-\beta(H_S' + \underbrace{\lambda}_{\text{book-keeping}} H_I' + H_B')}\right\}}{\mathrm{Tr}_{RC,B'}\left\{e^{-\beta(H_{RC} + \lambda H_I' + H_B')}\right\}} = \frac{\mathrm{Tr}_{RC}\left\{e^{-\beta H_S'}\right\}}{\mathrm{Tr}_{RC}\left\{e^{-\beta H_{RC}}\right\}} + \mathcal{O}(\lambda)$$

$$\bar{\rho}_S = \mathrm{Tr}_{RC}\left\{\bar{\rho}_S'\right\} = \frac{\mathrm{Tr}_{RC}\left\{e^{-\beta H_S'}\right\}}{\mathrm{Tr}_{S,RC}\left\{e^{-\beta H_S'}\right\}} = \frac{e^{-\beta H^*}\,\mathrm{Tr}_{RC}\left\{e^{-\beta H_{RC}}\right\}}{\mathrm{Tr}_{S,RC}\left\{e^{-\beta H_S'}\right\}}$$

$$= \dots \frac{e^{-\beta H^*}}{\mathrm{Tr}_S\left\{e^{-\beta H^*}\right\}} + \mathcal{O}(\lambda)$$

falls ME anwendbar auf SS $\longrightarrow$ red. SS vom glob. Gibbs-Zustand

nächste Woche: Vertretung S. Restrepo