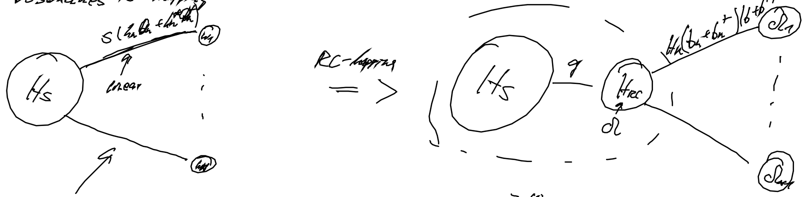
Wdh · bosonisches RC Mapping



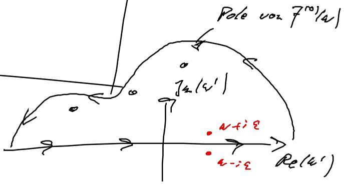
$J^{(0)}(\omega) = 2\pi \sum_k |h_k|^2 \delta(\omega - \omega_k)$

originale SD

$J^{(1)}(\omega) = 2\pi \sum_k |H_k|^2 \delta(\omega - \Omega_k)$

residuale SD

$$\boxed{\begin{aligned} \Omega_0^2 &= \frac{\int_0^\infty \omega J^{(0)}(\omega)\, d\omega}{\int_0^\infty \frac{J^{(0)}(\omega)}{\omega}\, d\omega} \qquad g^2 = \frac{1}{2\pi\Omega_0}\int \omega \cdot J^{(0)}(\omega) \cdot d\omega \\ J^{(1)}(\omega) &= \frac{4g^2 J^{(0)}(\omega)}{\left[\frac{1}{\pi}\mathcal{P}\int \frac{J^{(0)}(\omega')}{\omega-\omega'}\, d\omega'\right]^2 + \left[J^{(0)}(\omega)\right]^2} \end{aligned}}$$



$$\mathcal{P}\int_{-\infty}^{+\infty} \frac{J^{(0)}(\omega')}{\omega - \omega'}\, d\omega' = \lim_{\varepsilon \to 0} \int \frac{J^{(0)}(\omega')(\omega - \omega')}{(\omega - \omega')^2 + \varepsilon^2}$$

$\leftarrow [\omega - \omega' - i\varepsilon][\omega - \omega' + i\varepsilon]$

isolierte Pole

$$= 2\pi i \sum_k \underset{\omega' = \omega_k}{\mathrm{Res}} \frac{J^{(0)}(\omega')}{\omega - \omega} + \underbrace{2\pi i \lim_{\varepsilon \to 0} \underset{\omega' = \omega + i\varepsilon}{\mathrm{Res}} \frac{J^{(0)}(\omega')\overbrace{(\omega - \omega')}^{-i\varepsilon}}{\underbrace{[\omega - \omega' - i\varepsilon]}_{\to -2i\varepsilon}[\omega - \omega' + i\varepsilon]}}_{\frac{J^{(0)}(\omega)}{2}}$$

⇒ ME für Supersystem (System + RC + Kopplung)

$\dot{\rho} = \mathcal{L}\rho \rightsquigarrow \rho(t) = e^{\mathcal{L}t}\rho_0$ Markovsch

$\rho_S(t) = \mathrm{Tr}_{RC}\{\rho(t)\} \Rightarrow \mathrm{Tr}_{RC}\{\mathcal{L}\rho(t)\} = \dot{\rho}_S$

i.A. nicht-Markovsch

4.4. Fermionische RC Abbildung

4.4.1. Fermionische BTs

$C_k = \sum_q u_{kq} d_q + v_{kq} d_q^\dagger$

$\quad\;\; U \qquad\quad V$

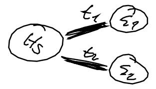
Erhalte Anti-Kommutator Relationen

$$\left.\begin{aligned} UU^\dagger + VV^\dagger &= \mathbb{1} \\ UV^T + VU^T &= 0 \end{aligned}\right\} \xrightarrow{V=0} U \text{ muss unitär sein für } V=0$$

hier: $V=0$ (keine Mischung)

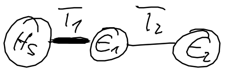
4.4.2. 2 Moden-Beispiel

$$H = H_S + (t_1 d^\dagger c_1 + t_1^* c_1^\dagger d) + (t_2 d^\dagger c_2 + t_2^* c_2^\dagger d) + \varepsilon_1 c_1^\dagger c_1 + \varepsilon_2 c_2^\dagger c_2$$



$$= [\ c_1 = u_{11} d_1 + u_{12} d_2 \quad c_2 = u_{21} d_1 + u_{22} d_2\ ]$$

$$= H_S + [d^\dagger \underbrace{(t_1 \cdot u_{11} + t_2 \cdot u_{21})}_{T_1} d_1 + d^\dagger \underbrace{(t_1 \cdot u_{12} + t_2 u_{22})}_{\overset{!}{=} 0} d_2 + h.c.]$$
$$+ \varepsilon_1 [\ldots][\ldots]^\dagger + \varepsilon_2 [\ldots][\ldots]$$



$$\boxed{U = \frac{1}{\sqrt{|t_1|^2 + |t_2|^2}} \begin{pmatrix} +t_1^* & -t_2 \\ +t_2^* & +t_1 \end{pmatrix}} \to T_1, \underbrace{T_2, \epsilon_1 / \epsilon_2}$$

$$T_1 = \sqrt{|t_1|^2 + |t_2|^2} \xrightarrow{t_i \to \infty} \infty$$

$$T_2 = \frac{t_1 \cdot t_2}{|t_1|^2 + |t_2|^2} (\varepsilon_2 - \varepsilon_1) \xrightarrow{t_i \to \infty} \text{bleibt endlich}$$

4.4.3. Ableitung der Abbildung

$$H = H_S + c \sum_k t_k^* c_k^\dagger - c^\dagger \sum_k t_k c_k + \sum_k \varepsilon_k c_k^\dagger c_k \quad \to \Gamma^{(0)}(\omega) = 2\pi \sum_k |t_k|^2 \delta(\omega - \varepsilon_k)$$

$$\overset{!}{=} H_S + \lambda c \underset{\substack{\uparrow \\ RC}}{d^\dagger} - \lambda c^\dagger d + \epsilon d^\dagger d + d \sum_k T_k^* d_k^\dagger - d^\dagger \sum_k T_k d_k + \sum_k \epsilon_k d_k^\dagger d_k$$

$$\to \Gamma^{(1)}(\omega) = 2\pi \sum_q |T_k|^2 \delta(\omega - \epsilon_k)$$

$$c_k = \sum_q u_{kq} d_q \qquad d = d_0$$

- Terme mit $c / c^\dagger \to$ Kopplung

$$\lambda \cdot d = \sum_k t_k \cdot c_k \qquad \lambda d^\dagger = \sum_k t_k^* c_k^\dagger \quad \Rightarrow \{\lambda d, \lambda d^\dagger\} = \lambda^2 = \sum_{kq} t_k t_q^* \underbrace{\{c_k, c_q^\dagger\}}_{\delta_{kq}}$$

$$\Rightarrow \lambda^2 = \sum_k |t_k|^2 = \boxed{\frac{1}{2\pi} \int_{-\infty}^{\infty} \Gamma^{(0)}(\omega)\, d\omega = \lambda^2} \text{ Kopplung System - RC}$$

- Energie

$$d = \sum_k \frac{t_k}{\lambda} c_k = \sum_k (U^\dagger)_{0k} c_k = \sum_k u_{k0}^* c_k \quad \rightsquigarrow u_{k0} = \frac{t_k^*}{\lambda}$$

$$\epsilon d^\dagger d = \sum_k \varepsilon_k \cdot |u_{k0}|^2 d^\dagger d \quad \Rightarrow \quad \epsilon = \sum_k \varepsilon_k \frac{|t_k|^2}{\lambda}$$

$$\Rightarrow \boxed{\epsilon = \frac{1}{2\pi\lambda^2} \int_{-\infty}^{\infty} \omega \cdot \Gamma^{(0)}(\omega)\, d\omega}$$

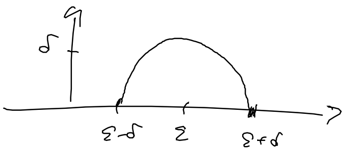
$$\boxed{\Gamma^{(1)}(\omega) = \frac{4\lambda^2\, \Gamma^{(0)}(\omega)}{\left[\frac{1}{\pi} \mathcal{P} \int_{-\infty}^{\infty} \frac{\Gamma^{(0)}(\omega')}{\omega - \omega'} d\omega'\right]^2 + \left[\Gamma^{(0)}(\omega)\right]^2}}$$

Unterschiede: g vs λ anders
Ω vs ϵ
$J(\omega) = -J(\omega)$ vs. $\Gamma(\omega) \geq 0\ \forall\omega$

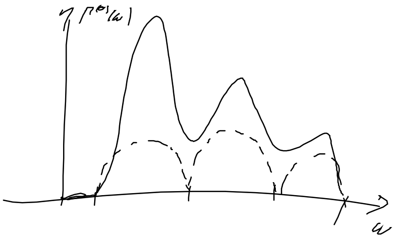
- $\Gamma^{(0)}(\omega) \to \alpha \cdot \Gamma^{(0)}(\omega) \longrightarrow \Gamma^{(1)}$ ist invariant

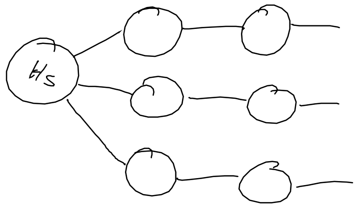
- rekursive Anwendung möglich
- $\Gamma(\omega) = \delta\sqrt{1-\left(\frac{\omega-\varepsilon}{\delta}\right)^2}\cdot\Theta\left(\delta^2-(\omega-\varepsilon)^2\right)$

nur für $\Gamma^{(0)}(\omega) \neq 0$ $\omega\in[\varepsilon-\delta, \varepsilon+\delta]$

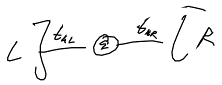


- Anwendung





4.4.4 SET (Beispiel)



$$H = \varepsilon d^\dagger d + \sum_\nu \sum_k \left(\varepsilon_{k\nu} c^\dagger_{k\nu} c_{k\nu} + t_{k\nu} d^\dagger c_{k\nu} + t^*_{k\nu} c^\dagger_{k\nu} d\right)$$

Breite

$$\Gamma_\nu^{(0)}(\omega) = \frac{\Gamma_\nu\, \delta_\nu^2}{(\omega-\varepsilon_\nu)^2+\delta_\nu^2}$$

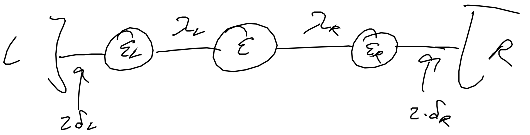
wideband: $\delta_\nu\to\infty$ $\Gamma_\nu^{(0)}(\omega) \Rightarrow \Gamma_\nu$

Höhe d. Maximums Position d. Max.

1 RC für jedes Reservoir

$$\Rightarrow H = \underbrace{\varepsilon d^\dagger d + \overset{\text{linke RC}}{\varepsilon_L d_L^\dagger d_L} + \overset{\text{rechte RC}}{\varepsilon_R d_R^\dagger d_R} + \left(\overset{\sqrt{\frac{\Gamma_L\delta_L}{2}}}{\lambda_L} d^\dagger d_L + \overset{\sqrt{\frac{\Gamma_R\delta_R}{2}}}{\lambda_R} d^\dagger d_R + h.c.\right)}_{H_S' = H_{TQD}}$$

$$+ \sum_\nu\sum_k \left[E_{k\nu} d^\dagger_{k\nu} d_{k\nu} + \left(T_{k\nu} d_\nu^\dagger d_{k\nu} + h.c.\right)\right]$$



$\Gamma_\nu^{(1)}(\omega) = 2\cdot\delta_\nu$ wird konstant

Vergleiche

i) exakte Lösung

$$I_M = \int d\omega\, T(\omega)\left[f_L(\omega) - f_R(\omega)\right]$$ geht zu berechnen

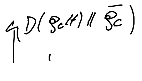
ii) naive ME für SET

$$I_M = \frac{\Gamma_L\Gamma_R}{\Gamma_L+\Gamma_R}\left[f_L(\varepsilon)-f_R(\varepsilon)\right] \xrightarrow{\Gamma_L=\Gamma_R=\Gamma} \frac{\Gamma}{2}\left[f_L(\varepsilon)-f_R(\varepsilon)\right]$$

iii) ME für TQD (supersystem): BAS

$\dot\rho = \mathcal{L}_{BAS}\,\rho$ Markovsch (weil $\mathcal{L}_{BAS}$ = Lindblad-Form) $\frac{d}{dt} D\left(\rho(t)\,\|\,\bar\rho\right) \le 0$

$\rho_C(t) = \mathrm{Tr}_{LR}\{\rho(t)\} = \begin{pmatrix}1-P_1(t) & 0\\ 0 & P_1(t)\end{pmatrix}$ $P_1(t) = \mathrm{Tr}\{d^\dagger d\,\rho(t)\}$



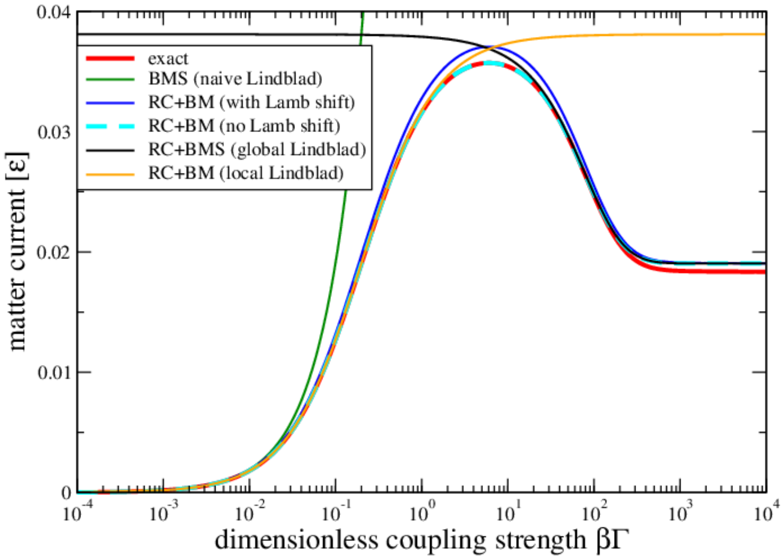
Markovsche Einbettung

nicht-Markovsche Entwicklung

iv.) nicht säkular -ME für TQD

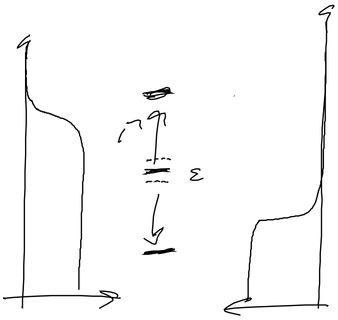
v.) lokale ME für Supersystem

$$\dot{\varrho} = -i\,[H_{TQD}, \varrho]$$
$$+ \Big[+ \Gamma_L^{(1)}(\varepsilon_L)[1-f_L(\varepsilon_L)] \big[d_L \varrho d_L^\dagger - \tfrac{1}{2}\{d_L^\dagger d_L, \varrho\}\big] + \Gamma_L^{(1)}(\varepsilon_L) \cdot f_L(\varepsilon_L)\ (d_L \leftrightarrow d_L^\dagger) \Big]$$
$$+ (L \to R)$$



$\mu_L \neq \mu_R$

$\Gamma_L = \Gamma_R = \Gamma$



4.5. Stationärer Zustand

$$\bar{\varrho}_S' = \frac{e^{-\beta H_S'}}{Z'} \qquad \text{Gibbs-Zustand des Supersystems}$$

$$\bar{\varrho}_S = \mathrm{Tr}_B\left\{ \frac{e^{-\beta(H_S + H_B + H_I)}}{Z} \right\} \xrightarrow[Z = Z_S \cdot Z_B]{H_I \to 0} \frac{e^{-\beta H_S}}{Z_S} \qquad (\text{lokaler Gibbs})$$

globaler Gibbs

Supersystem: $H_S' = H_S + H_{RC} + H_I$

$$e^{-\beta H^*} \equiv \frac{\mathrm{Tr}_B\{e^{-\beta(H_S + H_I + H_B)}\}}{\mathrm{Tr}_B\{e^{-\beta H_B}\}}$$

für $H_I \to 0$: $e^{-\beta H^*} = e^{-\beta H_S}$

Hamiltonian of mean force $\xrightarrow{H_I' \to 0} H^* \longrightarrow H_S$

"effektiver" System-Hamiltonian im Limes starker Kopplung

book-keeping

$$e^{-\beta H^*} = \frac{\text{Tr}_{RC,B'}\{e^{-\beta(H_S' + \lambda H_I' + H_B')}\}}{\text{Tr}_{RC,B'}\{e^{-\beta(H_{RC} + \lambda H_I' + H_B')}\}} = \frac{\text{Tr}_{RC}\{e^{-\beta H_S'}\}}{\text{Tr}_{RC}\{e^{-\beta H_{RC}}\}} + \mathcal{O}(\lambda)$$

$$\bar\rho_S = \text{Tr}_{RC}\{\bar\rho_S'\} = \frac{\text{Tr}_{RC}\{e^{-\beta H_S'}\}}{\text{Tr}_{S,RC}\{e^{-\beta H_S'}\}} = \frac{e^{-\beta H^*}\,\text{Tr}_{RC}\{e^{-\beta H_{RC}}\}}{\text{Tr}_{S,RC}\{e^{-\beta H_S'}\}}$$

$$= \ldots \frac{e^{-\beta H^*}}{\text{Tr}_S\{e^{-\beta H^*}\}} + \mathcal{O}(\lambda)$$

falls ME anwendbar auf SS ⟶ red. SS vom glob. Gibbs-Zustand

nächste Woche: Vertretung S. Restrepo