

English Summary:

Higher multipoles

$$\underline{A}^{(2)}(\underline{r}, t) = \frac{\mu_0}{4\pi} \left\{ \nabla \times \frac{1}{r} \underline{m}(t - \frac{r}{c}) + \frac{1}{6r^3} \dot{\underline{Q}}_r + \frac{1}{6cr^2} \ddot{\underline{Q}}_r \right\}$$

magnetic dipole radiation electric quadrupole radiation

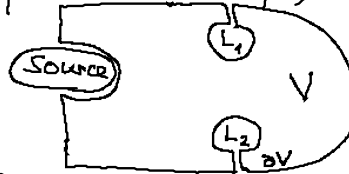
Wave optics and diffraction

$$\varphi(\underline{r}, t) = \varphi(\underline{r}) e^{-i\omega t}$$

Lorenz gauge $\Rightarrow (\Delta + k^2) \phi(\underline{r}) = -\frac{1}{\epsilon_0} \varrho(\underline{r}) \quad k = \frac{\omega}{c}$

(homog. wave eq. : Helmholtz eq.)

boundary value problem:



Scalar Kirchhoff identity

$$\phi(\underline{r}') = \int_{\partial V} d\Omega' \left[\tilde{G}(\underline{r}-\underline{r}') \nabla_{\underline{r}} \phi(\underline{r}) - \phi(\underline{r}) \nabla_{\underline{r}} \tilde{G}(\underline{r}-\underline{r}') \right] \quad \underline{r}' \in V, \underline{r} \in \partial V$$

(a) Green'sche Fkt. des unendl. Raumes:

Randbed. $\tilde{G}(\underline{r}-\underline{r}') \rightarrow 0$ für $r \rightarrow \infty$

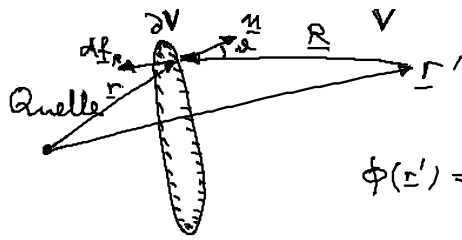
\Rightarrow retardiertes Pot. (S4.2)

$$G(\underline{r}-\underline{r}', t-t') = \begin{cases} \frac{1}{4\pi|\underline{r}-\underline{r}'|} \delta(t-t' - \frac{|\underline{r}-\underline{r}'|}{c}) & t-t' > 0 \\ 0 & t-t' < 0 \end{cases}$$

$$\tilde{G}(\underline{r}-\underline{r}') := \int_0^{\infty} d\tau G(\underline{r}-\underline{r}', \tau) e^{i\omega\tau} = \frac{e^{ik|\underline{r}-\underline{r}'|}}{4\pi|\underline{r}-\underline{r}'|} \quad \text{mit } k := \frac{\omega}{c}$$

$$\begin{aligned} \phi(\underline{r}, t) &= \int_{\mathbb{R}^3} d\underline{r}' \tilde{G}(\underline{r}-\underline{r}') e^{-i\omega t} \varrho(\underline{r}') / \epsilon_0 \\ &= \int_{\mathbb{R}^3} d\underline{r}' \frac{e^{i(k|\underline{r}-\underline{r}'| - \omega t)}}{4\pi|\underline{r}-\underline{r}'|} \varrho(\underline{r}') / \epsilon_0 \end{aligned}$$

beschreibt Überlagerung auslaufender Kugelwellen
(Ausstrahlbed.)



$$\underline{R} := \underline{r} - \underline{r}'$$

Kirchhoff-Identität:

$$\phi(\underline{r}') = \frac{1}{4\pi} \int_{\partial V} dA_{\underline{r}} \left\{ \frac{e^{ikR}}{R} \nabla_{\underline{r}} \phi(\underline{r}) - \phi(\underline{r}) \nabla_{\underline{r}} \frac{e^{ikR}}{R} \right\}$$

mit $dA_{\underline{r}} \frac{\underline{r} - \underline{r}'}{|\underline{r} - \underline{r}'|} = dA_{\underline{r}} \cos \alpha$

$$\frac{e^{ikR}}{R} \underbrace{\left(ik - \frac{1}{R} \right)}_{\frac{\underline{r} - \underline{r}'}{|\underline{r} - \underline{r}'|}}$$

und Beschränkung auf Fernzone von ∂V (d.h. $R \gg \frac{1}{k}$):

$$\phi(\underline{r}') \approx \frac{1}{4\pi} \int_{\partial V} dA_{\underline{r}} \left\{ \frac{\partial}{\partial n} \phi(\underline{r}) - ik \phi(\underline{r}) \cos \alpha \right\} \frac{e^{ikR}}{R}$$

richtungsabh. Amplitude Kugelwelle
 „Sekundärwelle“

Exakte Formulierung des Huygens'schen Prinzips

(Jeder Pkt. der Oberfläche eines Hindernisses ist Ausgangspkt. einer Kugelwelle; deren phasengerechte Überlagerung ergibt das Wellenfeld in \underline{r}' .)

(b) Greenfkt. zu Randbed. $\tilde{G}(\underline{r} - \underline{r}') \Big|_{\substack{\underline{r} \in \partial V \\ \underline{r}' \in V}} = 0$

$$\Rightarrow \phi(\underline{r}') = - \int_{\partial V} dA_{\underline{r}} \phi(\underline{r}) \nabla_{\underline{r}} \tilde{G}(\underline{r} - \underline{r}')$$

neue Greenfkt. $\tilde{G}(\underline{R}) = g(\underline{R}) + \frac{1}{4\pi} \frac{e^{ikR}}{R}$

$$(\Delta + k^2)g = 0 \quad \text{alte Greenfkt.} \\ \text{mit } g \Big|_{\partial V} = -\frac{1}{4\pi} \frac{e^{ikR}}{R} \rightarrow 0 \text{ für } r \rightarrow \infty$$

Beispiel für die Konstruktion von \tilde{G} :

Ebener Schirm

Spiegelladungsmethode:

$$\tilde{G}(\underline{r}-\underline{r}') = \frac{1}{4\pi} \left(\frac{e^{ik|\underline{r}-\underline{r}'|}}{|\underline{r}-\underline{r}'|} - \frac{e^{ik|\underline{r}-\underline{r}''|}}{|\underline{r}-\underline{r}''|} \right)$$

$$\nabla_{\underline{r}} \tilde{G}(\underline{r}-\underline{r}') = \frac{1}{4\pi} \left[\nabla_{\underline{r}} \frac{e^{ikR}}{R} - \nabla_{\underline{r}} \frac{e^{ikR''}}{R''} \right]$$

$$= \frac{1}{4\pi} \left[\frac{e^{ikR}}{R} \left(ik - \frac{1}{R} \right) \frac{\underline{r}-\underline{r}'}{|\underline{r}-\underline{r}'|} - \frac{e^{ikR''}}{R''} \left(ik - \frac{1}{R''} \right) \frac{\underline{r}-\underline{r}''}{|\underline{r}-\underline{r}''|} \right]$$

mit $R=R''$, $d\underline{f} \cdot \frac{\underline{r}-\underline{r}'}{|\underline{r}-\underline{r}'|} = -d\underline{f} \cdot \frac{\underline{r}-\underline{r}''}{|\underline{r}-\underline{r}''|} = d\underline{f} \cos \alpha$

folgt

$$d\underline{f} \cdot \nabla_{\underline{r}} \tilde{G} = d\underline{f} \frac{1}{2\pi} \frac{e^{ikR}}{R} \left(ik - \frac{1}{R} \right) \cos \alpha$$

Fernzone $\lambda \ll R$

Fernzone:

$$\phi(\underline{r}) = - \int_{\partial V} [d\underline{f} \cdot \nabla_{\underline{r}} \tilde{G}(\underline{r}-\underline{r}')] \phi(\underline{r}') = - \frac{i}{\lambda} \int_F d\underline{f} \phi(\underline{r}') \frac{e^{ik|\underline{r}-\underline{r}'|}}{|\underline{r}-\underline{r}'|} \cos \alpha$$

Randwerte $\phi(\underline{r}) \Big|_{\underline{r} \in F}$ ersates!

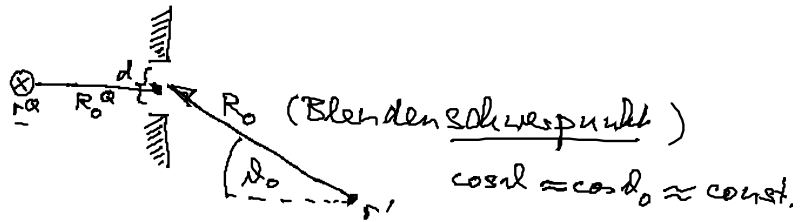
Annahme: $\phi(\underline{r}) \Big|_S = 0$ (Leiter)

$\phi(\underline{r}) \Big|_B = \frac{e^{ikR^Q}}{R^Q}$ freie einfallende Welle

$$\Rightarrow \phi(r') = -\frac{i}{\lambda} \int_B df \frac{e^{ik(R+R^Q)}}{R R^Q} \cos \alpha$$

$$\begin{aligned} R &= r - r' \\ R^Q &= r - r'^Q \\ df &= d^2 r \end{aligned}$$

Kleine Blende
(aber $\lambda \ll d$)



$$\phi(r') \approx -\frac{i}{\lambda} \frac{\cos \alpha_0}{R_0 R_0^Q} \int_B df e^{ik(R+R^Q)}$$

Grenzfälle:

(i) Fraunhofer'sche Beugung (Fernzone: $\lambda \ll d \ll R$)

Setze $R = R_0 + \xi$

$$R^2 \approx R_0^2 + 2R_0 \cdot \xi$$

$$R \approx R_0 + \frac{1}{2} \xi$$

$$\alpha := \frac{r_0}{R_0}$$



analog $R^Q \approx R_0^Q + \frac{1}{2} \xi$ $\alpha_0 := \frac{r_0^Q}{R_0^Q}$

$$\Rightarrow \phi(r') \approx -\frac{i}{\lambda} \frac{e^{ik(R_0+R_0^Q)}}{R_0 R_0^Q} \cos \alpha_0 \int_B d^2 s e^{ik(\alpha + \alpha_0) \cdot \xi}$$

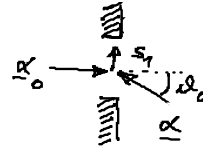
(ii) Fresnel'sche Beugung (Mittelzone $\lambda \ll R \approx d$)

$$R^2 = R_0^2 + 2R_0 \cdot \xi + \xi^2 \text{ nicht vernachlässigt!}$$

Beispiel: Fraunhofer'sche Beugung am Spalt
(1-dim.)

Senkrechter Einfall: $\alpha_0 \cdot \underline{s} = 0$

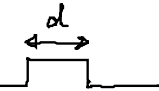
$$\begin{aligned} \phi(r') &= C \int_{-\frac{d}{2}}^{\frac{d}{2}} ds_1 e^{ik\alpha s_1} \\ &= \frac{C}{ik\alpha} \left(e^{ik\alpha \frac{d}{2}} - e^{-ik\alpha \frac{d}{2}} \right) \end{aligned}$$

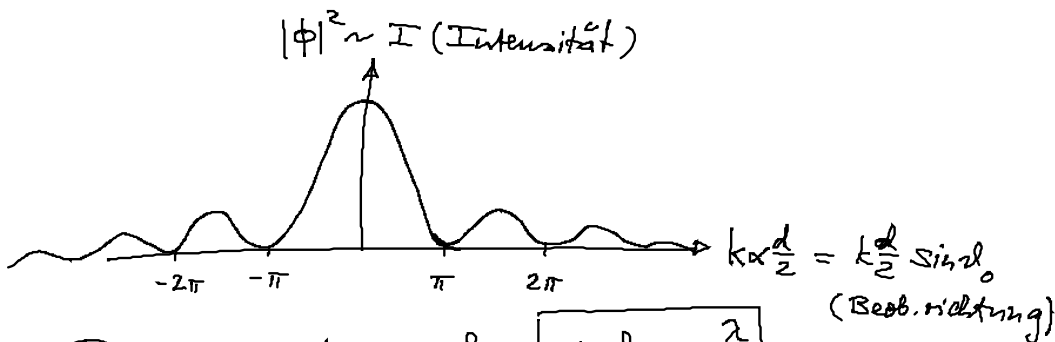


$$\begin{aligned} \alpha \cdot \underline{s} &= s_1 \sin \alpha_0 \\ \alpha &:= \sin \alpha_0 \end{aligned}$$

$$\boxed{\phi(r') = C d \frac{\sin(k\alpha \frac{d}{2})}{k\alpha \frac{d}{2}}}$$

"Spaltlicht"

Fouriertrafo von 



Beugungsminima bei $\boxed{\sin \alpha_0 = n \frac{\lambda}{d}}$

$$\frac{2\pi}{\lambda} \frac{d}{2} \cdot \sin \alpha_0 = n \pi$$