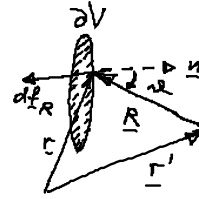


English Summary:

Huygens' Principle:

$$\phi(\underline{r}') = \frac{i}{4\pi} \int_{\partial V} d\underline{f}_R \left\{ \frac{\partial}{\partial n} \phi(\underline{r}) - ik\phi(\underline{r}) \cos \vartheta \right\} \frac{e^{ikR}}{R}$$

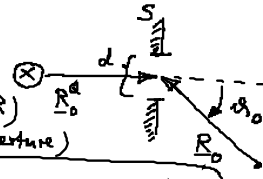
angle-dependent amplitude wave spherical wave



superposition of secondary waves emanating from obstacle

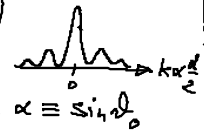
Planar screen S and aperture B:

Fraunhofer diffraction (far field $\lambda \ll d \ll R$)
($kR \gg 1$, small aperture)



$$\phi(\underline{r}') \approx -\frac{i}{\lambda} \frac{\cos \vartheta_0}{R_0 R_0^{\text{ap}}} \int_B d\underline{f} e^{ik(R+R^{\text{ap}})} \sim \frac{\sin(kx \frac{d}{2})}{kx \frac{d}{2}}$$

(Id est) $\alpha \equiv \sin \vartheta_0$



5. Materie in elektrischen u. magn. Feldern

5.1 Polarisation

Materie enthält mikroskop. el. geladene Bausteine
(Elektronen, Kerne, Ionen, etc.):

(i) freie Ladungsträger (Elektronen in Metallen, El. + Löcher in Halbleitern)

→ Beschleunigung in äußeren Feldern $\underline{E}, \underline{B}$:

$$\underline{F} = q(\underline{E} + \underline{v} \times \underline{B}) \rightarrow \text{el. Ströme}$$


→ Materialeigenschaft, durch Leitfähigkeit σ beschrieben

(ii) gebundene Ladungen (in Isolatoren)

→ Polarisierung im \underline{E} -Feld

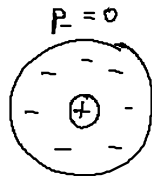
(a) Für $\underline{E} = 0$ vorhandene mikroskop. el. Dipole \underline{p} werden zur Minimierung der pot. Energie

$W_{el} = -\underline{p} \cdot \underline{E}$ vorzugsweise (gegen die zufällige therm. Bewegung) $\uparrow \uparrow \underline{E}$ orientiert.

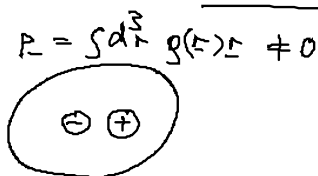
(z.B. H_2O )

(b) Nicht-polare Atome oder Moleküle werden

durch \underline{E} polarisiert \Rightarrow induzierte el. Dipole $\parallel \underline{E}$

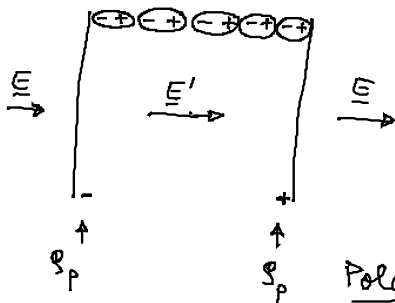


$\underline{E} = 0$



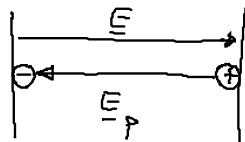
$\underline{E} \neq 0$

Makroshop. räumliche Mittelung



Polarisationsladungsdichte
(der gebundenen Ladungen)

\Rightarrow Gegenfeld \underline{E}_p gemäß $\epsilon_0 \nabla \cdot \underline{E}_p = \rho_p$



Resultierendes Gesamtfeld $\underline{E}' = \underline{E} + \underline{E}_p$

$$\epsilon_0 \nabla \cdot \underline{E}' = \epsilon_0 \underbrace{\nabla \cdot \underline{E}}_{\rho \text{ (freie Ladungsdichte)}} + \rho_p$$

$$\boxed{\epsilon_0 \nabla \cdot \underline{E}' = \rho + \rho_p}$$

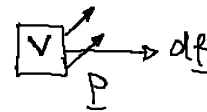
Polarisation $\underline{P}(\underline{r}, t) := -\epsilon_0 \underline{E}_p(\underline{r}, t)$ makroskop. lokales Feld, dessen Quellen Polarisationsladungen sind

$$\Rightarrow \begin{cases} \nabla \cdot (\epsilon_0 \underline{E}' + \underline{P}) = \rho \\ \nabla \cdot \underline{P} = -\rho_p \end{cases}$$

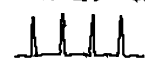
Die Verschiebung $\underline{D}(\underline{r}, t) := \epsilon_0 \underline{E}' + \underline{P}$ eff. Feldgröße, die als Quellen nur die freien Ladungen hat

$$\boxed{\nabla \cdot \underline{D} = \rho}$$

$\underline{P} \cdot d\underline{l} = dQ_p$ ist Polarisationsladung, die durch $d\underline{l}$ beim Übergang vom unpolarisierten zum polarisierten Zustand verschoben wird

$$\oint_{\partial V} \underline{P} \cdot d\underline{l} \stackrel{\text{Gauß}}{=} \int_V d\underline{r} \nabla \cdot \underline{P} = - \int_V d\underline{r} \rho_p$$


Zusammenhang mit den mikroskop. el. Dipolen

$$\rho_m(\underline{r}, t) = \sum_i q_i \delta(\underline{r} - \underline{r}_i(t)) \text{ mikroskop. Ladungsdichte}$$


$$\underline{P}_m(\underline{r}, t) = \sum_i \underline{p}_i(t) \delta(\underline{r} - \underline{r}_i) \text{ mikroskop. Dipoldichte}$$

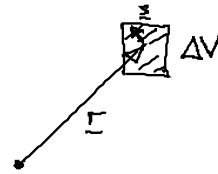
$$\int \underline{P}_m d^3r = \sum_i \underline{p}_i$$

Mittelung über kleine makroskop. Vol. ΔV :

$(\Delta V)^{1/3} \ll$ Längenskala der makroskop. Dichteveränderung

$$\underline{g}(\underline{r}, t) = \frac{1}{\Delta V} \int d^3s \underline{g}_m(\underline{r} + \underline{s}, t)$$

makroskop. Ladungsdichte



$$\underline{P}(\underline{r}, t) = \frac{1}{\Delta V} \int d^3s \underline{P}_m(\underline{r} + \underline{s}, t)$$

Polarisation $\stackrel{(*)}{\equiv}$ makroskop. Dipoldichte
(gemitteltes el. Dipolmoment pro Volumen)

Beweis von (*)

mikroskop. retardiertes Pot.:

$$\phi_m(\underline{r}, t) = \frac{1}{4\pi\epsilon_0} \int_{\mathbb{R}^3} d^3r' \frac{1}{|\underline{r} - \underline{r}'|} \rho_m(\underline{r}', t - \frac{|\underline{r} - \underline{r}'|}{c})$$

makroskop. gemitteltes Pot.:

$$\phi(\underline{r}, t) = \frac{1}{\Delta V} \int d^3s \phi_m(\underline{r} + \underline{s}, t)$$

$$= \frac{1}{4\pi\epsilon_0} \frac{1}{\Delta V} \int d^3s \int_{\mathbb{R}^3} d^3r' \frac{\rho_m(\underline{r}', t - \frac{|\underline{r} + \underline{s} - \underline{r}'|}{c})}{|\underline{r} + \underline{s} - \underline{r}'|}$$

$$\stackrel{\text{Subst. } \underline{r}'' = \underline{r}' - \underline{s}}{=} \frac{1}{4\pi\epsilon_0} \frac{1}{\Delta V} \int d^3s \int_{\mathbb{R}^3} d^3r'' \frac{\rho_m(\underline{r}'' + \underline{s}, t - \frac{|\underline{r} - \underline{r}''|}{c})}{|\underline{r} - \underline{r}''|}$$

$$= \frac{1}{4\pi\epsilon_0} \int_{\mathbb{R}^3} d^3r'' \frac{1}{|\mathbf{r}-\mathbf{r}''|} \underbrace{\frac{1}{\Delta V} \int_{\Delta V} d^3s \rho_m(\mathbf{r}''+\mathbf{s}, t - \frac{|\mathbf{r}-\mathbf{r}''|}{c})}_{\rho(\mathbf{r}'', t - \frac{|\mathbf{r}-\mathbf{r}''|}{c})}$$

makroskop. gemittelte Ladungsdichte

Analog:

mikroskop. Dipoldichte $\underline{P}_m(\mathbf{r}, t) = \sum_i p_i(t) \delta(\mathbf{r}-\mathbf{r}_i)$

mikroskop. Pot. der el. Dipole p_i (§ 4.3):

$$\begin{aligned} \phi_m(\mathbf{r}, t) &= -\frac{1}{4\pi\epsilon_0} \nabla_r \left\{ \sum_i \frac{1}{|\mathbf{r}-\mathbf{r}_i|} p_i \left(t - \frac{|\mathbf{r}-\mathbf{r}_i|}{c} \right) \right\} \\ &= -\frac{1}{4\pi\epsilon_0} \int_{\mathbb{R}^3} d^3r' \nabla_r \left\{ \frac{1}{|\mathbf{r}-\mathbf{r}'|} \underline{P}_m(\mathbf{r}', t - \frac{|\mathbf{r}-\mathbf{r}'|}{c}) \right\} \end{aligned}$$

makroskop. gemittelter el. Dipolpot.:

$$\begin{aligned} \phi(\mathbf{r}, t) &= \frac{1}{\Delta V} \int_{\Delta V} d^3s \phi_m(\mathbf{r}+\mathbf{s}, t) \\ &= -\frac{1}{4\pi\epsilon_0} \frac{1}{\Delta V} \int_{\Delta V} d^3s \int_{\mathbb{R}^3} d^3r' \nabla_r \left\{ \frac{\underline{P}_m(\mathbf{r}', t - \frac{|\mathbf{r}+\mathbf{s}-\mathbf{r}'|}{c})}{|\mathbf{r}+\mathbf{s}-\mathbf{r}'|} \right\} \end{aligned}$$

$$\begin{aligned} \mathbf{r}'' &= \mathbf{r} - \mathbf{s} \\ &= -\frac{1}{4\pi\epsilon_0} \int_{\mathbb{R}^3} d^3r'' \nabla_r \left\{ \frac{1}{|\mathbf{r}-\mathbf{r}''|} \underline{P}_m(\mathbf{r}'', t - \frac{|\mathbf{r}-\mathbf{r}''|}{c}) \right\} \end{aligned}$$

makroskop. Dipoldichte

Umformung:

$$\nabla_r \left\{ \frac{1}{|\mathbf{r}-\mathbf{r}'|} \underline{P}_m(\mathbf{r}', t - \frac{|\mathbf{r}-\mathbf{r}'|}{c}) \right\} = -\nabla_{r'} \left\{ \frac{1}{|\mathbf{r}-\mathbf{r}'|} \underline{P}_m(\mathbf{r}', t - \frac{|\mathbf{r}-\mathbf{r}'|}{c}) \right\}$$

$$\Rightarrow \phi(\underline{r}, t) = \underbrace{\frac{1}{4\pi\epsilon_0} \int_{\mathbb{R}^3} d^3r' \nabla_{\underline{r}'} \{ \dots \}}_{=0 \text{ (Gauß)}} + \frac{1}{4\pi\epsilon_0} \int_{\mathbb{R}^3} d^3r' \frac{1}{|\underline{r}-\underline{r}'|} \underbrace{\left[-\nabla_{\underline{r}'} \underline{P}(\underline{r}', t') \right]}_{\rho_p(\underline{r}', t - \frac{|\underline{r}-\underline{r}'|}{c})} \Big|_{t' = t - \frac{|\underline{r}-\underline{r}'|}{c}}$$

makroskop. Pot. einer
Ladungsdichte ρ_p

$$\rho_p(\underline{r}, t) = -\nabla_{\underline{r}} \underline{P}(\underline{r}, t)$$

↓
Polarisation $\underline{P} := -\epsilon_0 \underline{E}_p$