

English Summary:

Dielectric dispersion

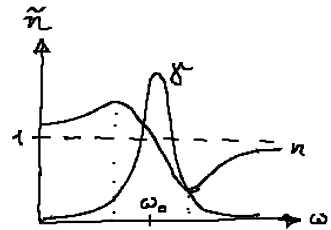
$$\hat{\underline{P}}(\omega) = \epsilon_0 \hat{\chi}(\omega) \hat{\underline{E}}(\omega)$$

$$\hat{\chi}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dt \chi(t) e^{i\omega t}$$

$$\underline{P}(t) = \frac{\epsilon_0}{\sqrt{2\pi}} \int_{-\infty}^t dt' \chi(t-t') \underline{E}(r, t')$$

dielectric fct. $\epsilon(\omega) = \epsilon'(\omega) + i\epsilon''(\omega)$

(group velocity $v_g = \frac{d\omega}{dk'} = \frac{c}{n + \omega \frac{dn}{d\omega}}$) ($\chi > 0$)



Kramers-Kronig:

(follows from causality:

$\chi(t) = 0$ for $t < 0$)

$$\epsilon'(\omega) - 1 = \frac{1}{\pi} \int_{-\infty}^{\infty} d\omega' \frac{\epsilon''(\omega')}{\omega' - \omega}$$

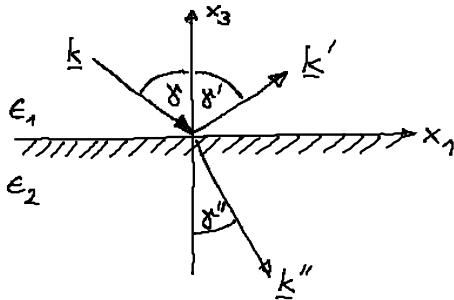
$$\epsilon''(\omega) = -\frac{1}{\pi} \int_{-\infty}^{\infty} d\omega' \frac{\epsilon'(\omega') - 1}{\omega' - \omega}$$

anomalous dispersion

5.7 Brechung und Reflexion

Wellenausbreitung in geschichteter Medien:

(transparent $\Rightarrow \epsilon_i \in \mathbb{R}, i=1,2$)



$$\frac{\omega}{c_1} = |k| = |k'| = \frac{\omega'}{c_1}$$

$$|k''| = \frac{\omega''}{c_2}$$

$$c_i = \frac{c}{n_i} = \frac{c}{\sqrt{\epsilon_i}} \quad i=1,2$$

Einfallende Welle $\underline{E} = \underline{E}_0 e^{i(k \cdot r - \omega t)}$
 Reflektierte Welle $\underline{E}' = \underline{E}'_0 e^{i(k' \cdot r - \omega' t)}$
 Transmittierte Welle $\underline{E}'' = \underline{E}''_0 e^{i(k'' \cdot r - \omega'' t)}$

Grenzbedingungen für Felder

$$\begin{aligned} \underline{n} \times (\underline{E}^{(1)} - \underline{E}^{(2)}) &= 0 && \text{Tang. komp. v. } \underline{E} \text{ stetig} \\ \underline{n} \cdot (\underline{D}^{(1)} - \underline{D}^{(2)}) &= \sigma && \text{Norm. komp. v. } \underline{D} = \epsilon_0 \underline{E} \text{ stetig} \\ \underline{n} \times (\underline{H}^{(1)} - \underline{H}^{(2)}) &= \underline{j} && \text{Tang. komp. v. } \underline{H} \text{ stetig} \\ \underline{n} \cdot (\underline{B}^{(1)} - \underline{B}^{(2)}) &= 0 && \text{Normalkomp. v. } \underline{B} = \mu_0 \underline{H} \text{ stetig} \end{aligned}$$

Grenzbed. für \underline{E} (linear polarisiert)

$$\left. \underline{E}_1 + \underline{E}'_1 \right|_{x_3=0} = \left. \underline{E}''_1 \right|_{x_3=0} \quad \text{Tang. komp. stetig}$$

$$t=0: E_{01} e^{-i\omega t} + E'_{01} e^{-i\omega' t} = E''_{01} e^{-i\omega'' t} \Rightarrow \begin{cases} \omega = \omega' = \omega'' \\ E_{01} + E'_{01} = E''_{01} \end{cases}$$

$$t=0: E_{01} e^{ik_1 x_1} + E'_{01} e^{ik'_1 x_1} = E''_{01} e^{ik''_1 x_1} \Rightarrow \begin{cases} k_1 = k'_1 = k''_1 \end{cases}$$

$$\Rightarrow \underbrace{|k|}_{\omega/c_1} \sin \gamma = \underbrace{|k'|}_{\omega/c_1} \sin \gamma' = \underbrace{|k''|}_{\omega/c_2} \sin \gamma''$$

$$\Rightarrow \begin{cases} \sin \gamma = \sin \gamma' \\ \frac{\sin \gamma''}{\sin \gamma} = \frac{c_2}{c_1} = \frac{n_1}{n_2} \end{cases}$$

Reflexionsgesetz

Brechungsgesetz
(Snellius)

Bestimmung der Amplituden:

(a) Polarisation von $\underline{E} \perp$ Einfallsebene

$$E_{01} = E'_{01} = E''_{01} = 0$$

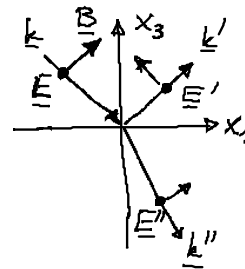
$$E_{03} = E'_{03} = E''_{03} = c$$

$$(1) \quad \boxed{E_{02} + E'_{02} = E''_{02}} \quad \text{Tang. komp.}$$

Mit $\underline{B}_0 = \frac{c}{\omega} (\underline{k} \times \underline{E}_0) = \frac{c}{\omega} E_{02} \begin{pmatrix} -k_3 \\ 0 \\ k_1 \end{pmatrix}$ folgt für Tang. komp. v. \underline{B} :

$$B_{01} + B'_{01} = B''_{01} \Rightarrow k_3 E_{02} + k'_3 E'_{02} = k''_3 E''_{02}$$

$$\text{Reflexionsgesetz} \Rightarrow k_3 = -k'_3 \Rightarrow \boxed{k_3 (E_{02} - E'_{02}) = k''_3 E''_{02}} \quad (2)$$



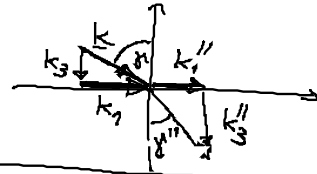
$$(1) \text{ in } (2) \Rightarrow k_3 (E_{02} - E'_{02}) = k_3'' (E_{02} + E'_{02}) \Rightarrow (k_3 - k_3'') E_{02} = (k_3 + k_3'') E'_{02}$$

$$\Rightarrow \boxed{\frac{E'_{02}}{E_{02}} = \frac{k_3 - k_3''}{k_3 + k_3''}, \quad \frac{E''_{02}}{E_{02}} = \frac{2k_3}{k_3 + k_3''}}$$

Drücke k_3'' durch Brechungswinkel γ'' aus!

$$\Rightarrow k_3'' = |k_3''| \cos \gamma'' = |k_3| \underbrace{\left(\frac{n_2}{n_1}\right)}_{\frac{\sin \gamma}{\sin \gamma''}} \cos \gamma''$$

$$k_3 = |k_3| \cos \gamma \quad \frac{\sin \gamma}{\sin \gamma''}$$



$$\Rightarrow \boxed{\frac{E'_{02}}{E_{02}} = \frac{\cos \gamma \sin \gamma'' - \sin \gamma \cos \gamma''}{\cos \gamma \sin \gamma'' + \sin \gamma \cos \gamma''} = \frac{\sin(\gamma'' - \gamma)}{\sin(\gamma'' + \gamma)}}$$

$$\boxed{\frac{E''_{02}}{E_{02}} = \frac{2 \sin \gamma'' \cos \gamma}{\sin(\gamma'' + \gamma)}}$$

Fresnel'sche Formeln

Intensitätsverhältnisse

Zeitmittel des Poyntingvektors $\langle \underline{S} \rangle = \frac{1}{T} \int_0^T dt \underline{E} \times \underline{H} \sim |\underline{E}_0|^2$

Reflexionskoeff.:

$$\boxed{R_{\perp} = \left| \frac{E'_{02}}{E_{02}} \right|^2 = \frac{\sin^2(\gamma'' - \gamma)}{\sin^2(\gamma'' + \gamma)}}$$

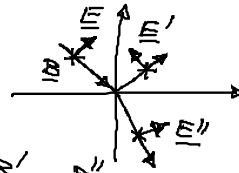
⊥
polarisiert

Transmissionskoeff.:

$$\boxed{T_{\perp} = \left| \frac{E''_{02}}{E_{02}} \right|^2 = \frac{4 \sin^2 \gamma'' \cos^2 \gamma}{\sin^2(\gamma'' + \gamma)} = 1 - R_{\perp}}$$

(b) Polarisation von $\underline{E} \parallel$ Einfallsebene

$\underline{B} \perp$ Einfallsebene



\Rightarrow analoge Argumentation für $B_{02}, B'_{02}, B''_{02}$
wie in (a)

$$\Rightarrow \frac{E'_{\parallel}}{E_{\parallel}} = \frac{\tan(\alpha - \alpha'')}{\tan(\alpha + \alpha'')}, \quad \frac{E''_{\parallel}}{E_{\parallel}} = \frac{2 \sin \alpha'' \cos \alpha}{\sin(\alpha + \alpha'') \cos(\alpha'' - \alpha)}$$

$$R_{\parallel} = 1 - T_{\parallel} = \frac{\tan^2(\alpha'' - \alpha)}{\tan^2(\alpha'' + \alpha)}$$

Bem.: (i) Bei Reflexion und Brechung wird i.a. die Polarisationsrichtung gedreht!

Speziell für:

$$\alpha'' + \alpha = \frac{\pi}{2} \Rightarrow \tan(\alpha'' + \alpha) \rightarrow \infty \Rightarrow \boxed{R_{\parallel} = 0}$$

\Rightarrow reflektierte Welle ist vollständig polarisiert
 \perp Einfallsebene

$\alpha =$ Brewster-Winkel α_B mit $\tan \alpha_B = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$

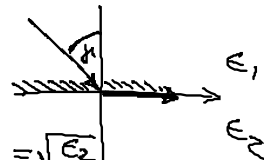
denn:

$$\tan \alpha = \frac{\sin \alpha}{\cos \alpha} = \frac{\sin \alpha}{\sin(\frac{\pi}{2} - \alpha)} = \frac{\sin \alpha}{\sin \alpha''} = \frac{n_2}{n_1} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

$$\begin{array}{c} \nearrow \sin \alpha \\ \searrow \cos \alpha = \sin(\frac{\pi}{2} - \alpha) \end{array}$$

(ii) Totalreflexion

Sei $\epsilon_2 < \epsilon_1 \Rightarrow$

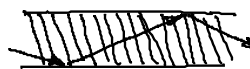


für $\alpha = \alpha_c$ mit $\sin \alpha_c = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$

(Grenzwinkel der Totalreflexion)

$$\alpha'' = \frac{\pi}{2}, \quad R_{\perp} = R_{\parallel} = 1$$

$$T_{\perp} = T_{\parallel} = 0$$



für $\eta > \eta_c \Rightarrow k_3'' = \frac{i}{d}$ imaginär

evaneszente Welle $\underline{E}'' = \underbrace{\underline{E}_0'' e^{-|x_3|/d}}_{\text{evaneszent}} e^{i(k_1'' x_1 - \omega t)}$