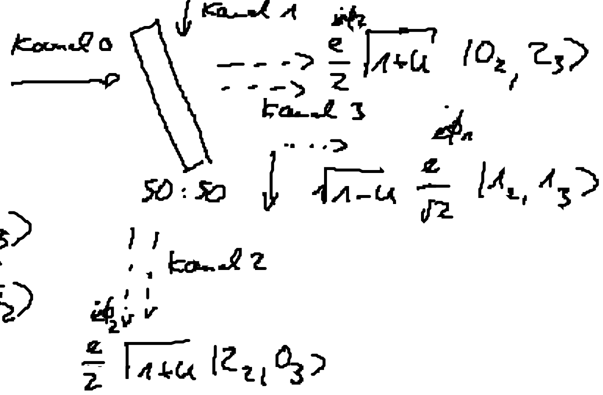
Ununterscheidbare Photonen. Beispiel

(Fachwort: „indistinguishability")

$$|\Psi_{in}\rangle = \int dt \int dt' \, \beta(t,t') \, a_0^\dagger(t) \, a_1^\dagger(t') \, |vac\rangle$$



Messgröße: $g^{(2)}(0) = \dfrac{\langle a_3^\dagger a_2^\dagger a_2 a_3\rangle}{\langle a_3^\dagger a_3\rangle\langle a_2^\dagger a_2\rangle}$

$g^{(2)}(0) = 0$ wenn $\boxed{u = 1}$

$$u = Re\left[\int dt \int dt' \, \beta(t,t') \, \beta^*(t',t)\right]$$

(1.) $\langle\Psi_{in}|\Psi_{in}\rangle = 1 = \langle\Psi_{out}|\Psi_{out}\rangle$

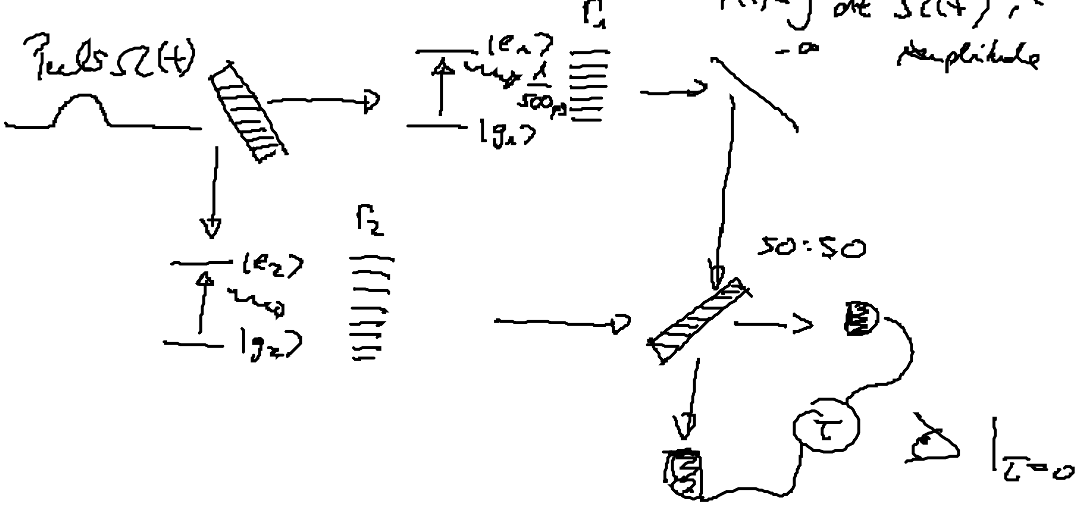
(2.) $\int dt_1 \int dt_2 \; |\beta(t_1,t_2)|^2 = 1 = \int dt_1 \int dt_2 \, |\beta(t_2,t_1)|^2$

(3.) $|\Psi_{in}\rangle = \int dt' \int dt'' \; \beta(t',t'') \; a_0(t') \, a_1(t'') \, |vac\rangle$

Beispiel:

$F(t) = \int_{-\infty}^{t} dt' \, \tilde{\Omega}(t')$, $F(\infty) = \pi$

Amplitude



der Puls ist so schnell, dass für beide Emitter gilt

$$\frac{\Omega(t)}{\Gamma_{1,2}} \gg 1 \quad \text{und d.h.} \quad |g_1\rangle \xrightarrow{\Omega} |e_1\rangle$$
$$|g_2\rangle \xrightarrow{\Omega} |e_2\rangle$$

also das Reservoir kann nicht reagieren.
Wir müssen also nur die Dynamik des Zwei-Niveau-System mit Anfangsbedingungen lösen $\underline{c_{e_1}(0)=1}$, $\underline{c_{e_2}(0)=1}$

$$|\psi(t)\rangle = |\psi(t)\rangle_1 \otimes |\psi(t)\rangle_2$$

$$|\psi(t)\rangle_i = \underbrace{c_{e_i}(t)\,|e_i, \text{vac}\rangle}_{\text{Anregung im Emitter}} + \underbrace{\int d\omega\; c_{g_i}^{\omega}(t)\,|g_i, \ldots 1_{\omega_i} \ldots\rangle}_{\text{Anregung im Reservoir}}$$

$$H = \sum_{i=1,2} \hbar\omega_e^i\, |e_i\rangle\langle e_i| + \hbar g_i \int d\omega \left[a_\omega^i\, |e_i\rangle\langle g_i| + a_\omega^{i+}\, |g_i\rangle\langle e_i|\right]$$
$$+ \sum_{i=1,2} \int d\omega_i\; \hbar\omega_i\; a_{\omega_i}^+ a_{\omega_i}$$

Schrödingergleichung: $i\hbar \frac{d}{dt}|\psi(t)\rangle = H|\psi(t)\rangle$

gehe ins Wechselwirkungsbild

$$U(t,0) = \exp\left[it\left(\sum_{i=1,2} \omega_e^i\, |e_i\rangle\langle e_i| + \int d\omega_i\; \omega_i\, a_{\omega_i}^+ a_{\omega_i}\right)\right]$$

$$i\hbar \frac{d}{dt}|\psi\rangle_I = \left[U H U^+ - i\hbar U \frac{d}{dt} U^+\right] |\psi(t)\rangle_I$$
$$= \left[\sum_i \hbar g_i \int d\omega_i\; e^{-i(\omega_i - \omega_e^i)t}\, a_\omega^i\, |e_i\rangle\langle g_i| + \text{h.c.}\right] |\psi(t)\rangle_I$$

$$i\hbar \langle e_j| \frac{d}{dt}|\psi_I\rangle = i\hbar\, \dot{c}_{e_j} = \hbar g_j \int d\omega_j'\; e^{-i(\omega_j' - \omega_e^j)t}\, c_{g_j}^{I\omega'}(t)$$

$$i\hbar\, \dot{c}_{g_j}^{I\omega'} = \hbar g_j\, e^{i(\omega_j' - \omega_e^j)t}\, c_{e_j}^I(t)$$

$\underline{\text{Löse}}$: $$c_{g_j}^{I\omega'}(t) - \underbrace{c_{g_j}^{I\omega'}(0)}_{=0} = \int_0^t dt'\,(-i g_j)\, e^{i(\omega_j' - \omega_e^j)t'}\, c_{e_j}^I(t')$$

Treiparent

$$\dot{c}^I_{e_j}(t) = (-i g_j)^2 \int d\omega_j' \, e^{-i(\omega_j' - \omega_e^j)t} \int_0^t dt' \, e^{i(\omega_j' - \omega_j)t'} \, c^I_{e_j}(t')$$

$$2\pi\,\delta(t-t') \quad | \text{ mit } \int_0^\infty d\omega \to \int_{-\infty}^{+\infty} d\omega$$

Narrow bandwidth approximation

$$= -g_j^2\, 2\pi \int_0^t dt' \, \delta(t-t')\, e^{-i\omega_{a_j}(t'-t)}\, c^I_{e_j}(t')$$

Randauswerten $\int_0^t dt' f(t')\,\delta(t-t') \overset{!}{=} \frac{1}{2} f(t)$

$$\dot{c}^I_{e_j} = -\underbrace{g_j^2\,\pi}_{\Gamma_j}\; c^I_{e_j}(t) \qquad \boxed{c^I_{e_j}(t) = \underbrace{c^I_{e_j}(0)}_{=1}\, e^{-\Gamma_j t}}$$

$$c^{I\omega'}_{g_j}(t) = \int_0^t dt' \,(-i g_j)\, e^{i(\omega_j' - \omega_e^j)t' - \Gamma_j t'}$$

$t \to \infty$ so, dass der Zerfallsprozess sicher beendet, $c^I_{e_j}(\infty) \to 0$

$$|\psi(\infty)\rangle_j = \int d\omega_j \, c^\infty_{g_j}(\infty)\, |g_j \dots 1_{\omega_j} \dots\rangle$$

$$= \int d\omega_j \int_0^\infty dt_j \underbrace{(-i g_j)\, e^{i(\omega_j - \omega_e^j)t_j - \Gamma_j t_j}}_{f(t_j,\omega_j)} \, |g_j, \dots 1_{\omega_j} \dots\rangle$$

also

$$|\psi(\infty)\rangle_{12} = \int d\omega_1 \int dt_1 \int d\omega_2 \int dt_2 \, f(t_1,\omega_1)\, f(t_2,\omega_2)\, |g_1 g_2, 1_{\omega_1} 1_{\omega_2}\rangle$$

$$= \int dt_1 \int dt_2 \Big[-g_1 g_2 \int d\omega_1 \int d\omega_2 \, e^{-i\omega_e^1 t_1 - i\omega_e^2 t_2 - \Gamma_1 t_1 - \Gamma_2 t_2}\, e^{i\omega_1 t_1}\, e^{i\omega_2 t_2}\, a^\dagger_{\omega_1} a^\dagger_{\omega_2} \Big] |vac\rangle$$

$$\overset{!}{=} \int dt_1 \int dt_2 \, \beta(t_1,t_2)\, a_1^\dagger(t_1)\, a_2^\dagger(t_2)\, |vac\rangle$$

zu zeigen: $[a_1(t_1), a_1^+(t_2)] = \delta(t_1 - t_2)$

$$\int d\omega_1 e^{-i\omega_1 t_1} a_{\omega_1} \int d\omega_2 e^{i\omega_2 t_2} a_{\omega_2}^+$$

$$= \int d\omega_1 \int d\omega_2 \, e^{-i\omega_1 t_1 + i\omega_2 t_2} \left[\delta(\omega_1 - \omega_2) + a_{\omega_2}^+ a_{\omega_1}\right]$$

aus Hamilton-Operator Definition; Schrödingerbild

$$[a_1(t_1), a_1^+(t_2)] = 2\pi \delta(t_1 - t_2)$$

$$\beta(t_1, \beta t_2) = -2\pi g_1 g_2 e^{-i\omega_e^1 t_1 - \Gamma_1 t_1 - i\omega_e^2 t_2 - \Gamma_2 t_2}$$

$$a_1^+(t_1) = \int d\omega_1 \, e^{i\omega_1 t_1} \frac{a_{\omega_1}^+}{\sqrt{2\pi}}$$

$$\int dt_1 \int dt_2 \, |\beta(t_1, t_2)|^2 = 1$$

$$\langle \Psi_{in} | \Psi_{in} \rangle = \left| \int dt_1 \int dt_2 \, \beta(t_1, t_2) a_1^+(t_1) a_2^+(t_2) |vac\rangle \right|^2$$

$$= \int dt_1 \int dt_2 \, |\beta(t_1, t_2)|^2$$

$$= \underbrace{4\pi^2 g_1^2 g_2^2}_{4\Gamma_1 \Gamma_2} \underbrace{\int_0^\infty dt_1 \int_0^\infty dt_2 \, e^{-2\Gamma_1 t_1 - 2\Gamma_2 t_2}}_{= \frac{[0-1]}{-2\Gamma_1} \frac{[0-1]}{-2\Gamma_2}}$$

$= 1$, also $\beta(t_1, t_2)$ sinnvoll eingeführt

wie sieht nun U aus? $U = \mathrm{Re}[A]$

$$A = \int dt_1 \int dt_2 \, \beta(t_1, t_2) \beta^*(t_2, t_1)$$

$$= \int dt_1 \int dt_2 \, (2\pi g_1 g_2)^2 \exp\left[i\omega_e^1 t_1 - \Gamma_1 t_1 + i\omega_e^2 t_2 - \Gamma_2 t_2 - i\omega_e^1 t_2 - \Gamma_1 t_2 - i\omega_e^2 t_1 - \Gamma_2 t_1\right]$$

$$= 4\Gamma_1\Gamma_2 \int dt_1 \exp[(i\omega_e^1 - i\omega_e^2 - \Gamma_1 - \Gamma_2)t_1] \int dt_2 \exp[(i\omega_e^2 - i\omega_e^1 - \Gamma_1 - \Gamma_2)t_2]$$

$$= 4\Gamma_1\Gamma_2 \frac{(0-1)}{i(\omega_e^1-\omega_e^2)-(\Gamma_1+\Gamma_2)} \frac{(0-1)}{i(\omega_e^2-\omega_e^1)-(\Gamma_1+\Gamma_2)}$$

$$= 4\Gamma_1\Gamma_2 \frac{-i(\omega_e^1-\omega_e^2)-(\Gamma_1+\Gamma_2)}{(\omega_e^1-\omega_e^2)^2+(\Gamma_1+\Gamma_2)^2} \frac{-i(\omega_e^2-\omega_e^1)-(\Gamma_1+\Gamma_2)}{(\omega_e^2-\omega_e^1)^2+(\Gamma_1+\Gamma_2)^2}$$

$$U = Re[A] = 4\Gamma_1\Gamma_2 \frac{(\Gamma_1+\Gamma_2)^2-(\omega_e^1-\omega_e^2)^2}{[(\omega_e^1-\omega_e^2)^2+(\Gamma_1+\Gamma_2)^2]^2}$$

was passiert, wenn $\omega_e^1 = \omega_e^2$, $\Gamma_1 = \Gamma_2 = \Gamma$ $(g_1 = g_2)$

$$U = 4\Gamma^2 \frac{(2\Gamma)^2}{[0-(2\Gamma)^2]^2} = \frac{16\Gamma^4}{16\Gamma^4} = 1,$$ perfekter Überlapp

$$\Delta_{12} = \omega_e^1 - \omega_e^2 = 0$$

$$Re[A] = \frac{4\Gamma_1\Gamma_2}{(\Gamma_1+\Gamma_2)^2} \frac{1-\left(\frac{\Delta_{12}}{\Gamma_1+\Gamma_2}\right)^2}{\left[\left(\frac{\Delta_{12}}{\Gamma_1+\Gamma_2}\right)^2+1\right]} \overset{(\Delta_{12}=0)}{=} \frac{4\Gamma_1+\Gamma_2}{(\Gamma_1+\Gamma_2)^2}$$

$$\Gamma_1 = \alpha\Gamma, \quad \Gamma_2 = \Gamma$$

$$\alpha > 0$$

$$U = \frac{4\Gamma^2\alpha}{\Gamma^2(\alpha+1)^2} = 4\frac{\alpha}{(\alpha+1)^2} \xrightarrow{\text{wenn } \alpha \ll 1} 0$$

$\Rightarrow$ Korrelationsmessung hängt ab von U,
und U ist ein direktes Maß für
die Ähnlichkeit der Emitter!! (α, Δ_{12})