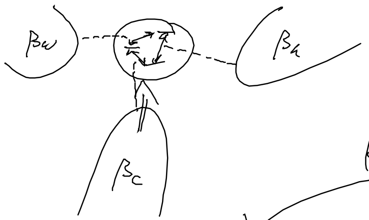
Fr. 5.7. $8^{00}-10^{00}$ EB 301

Nachholausgleich 8.7. 8^{00} EW 705

Wdh • 3-Terminal Kältemaschine



$$\boxed{m\ddot{x} + \gamma \dot{x} + \frac{\partial V}{\partial x} = f_{stoch}(t)}$$

• Fokker-Planck-Gleichung unterdämpft

+ $\partial_t P = \left\{ \frac{\gamma}{m} - v\cdot\partial_x + \left(\frac{1}{m}\frac{\partial V}{\partial x} + \frac{\gamma}{m}\cdot v\right)\partial_v + D\,\partial_v^2 \right\} P(x,v,t)$

+ $\bar{P} \propto e^{-\beta\left[\frac{1}{2}m v^2 + V(x)\right]}$

+ $V(x) = \frac{1}{2} k x^2$

+ 1. HS: $\frac{d}{dt}\langle E \rangle = \frac{d}{dt}\langle \frac{1}{2} m v^2 + \frac{1}{2} k x^2 \rangle = I_E(t)$

+ 2. HS: $\frac{d}{dt} S_{univ.} = \frac{d}{dt} S_{System} + \left(-\beta I_E(t)\right) \geq 0$

• analog überdämpft : $m\ddot{x} \approx 0$

4.2.3. Verbindung Ratengleichung

• überdämpft: $x \longrightarrow x_i$ $\qquad E_i = V(x_i) = V_i$

$P(x,t) \longrightarrow P(x_i,t) = P_i(t)$

$\partial_x P(x,t) \longrightarrow \frac{P_{i+1}(t) - P_{i-1}(t)}{2\Delta x} \rightsquigarrow \frac{P_{i+1/2}(t) - P_{i-1/2}(t)}{\Delta x}$, $\quad \frac{P(x+\frac{\Delta x}{2}) - P(x-\frac{\Delta x}{2})}{\Delta x}$

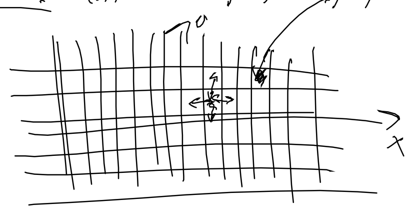
$\dot{P}_i = \sum_j W_{ij} P_j \qquad \Delta V = V_{i+1} - V_i$

$\partial_x^2 P(x,t) \longrightarrow \frac{P_{i+1}(t) - 2P_i(t) + P_{i-1}(t)}{\Delta x^2}$

$\frac{W_{i,i+1}}{W_{i+1,i}} = 1 + \frac{\Delta V}{k_B T} + \mathcal{O}(\Delta V^2) = e^{\beta \Delta V} + \mathcal{O}(\Delta V^2)$

$\partial_x\left[\partial_x P(x)\right] = \frac{P'(x+\frac{\Delta x}{2}) - P'(x-\frac{\Delta x}{2})}{\Delta x} = \frac{P(x+\Delta x) - P(x) - \left[P(x) - P(x-\Delta x)\right]}{\Delta x^2}$

unterdämpft : $P(x,t) \to P(x_i, v_j, t) = P_{ij}(t)$



4.3. Quanten-Otto-Prozess

• klassisch : 2 Adiabaten & 2 Isochoren

· jetzt HO

$$H(t) = \frac{p^2}{2m} + \frac{1}{2} m \omega^2(t) x^2 \qquad x = \sqrt{\frac{\hbar}{2m\omega(t)}}\left[a^+(t) + a(t)\right]$$

$$= \hbar\omega(t)\left(a^+(t)a(t) + 1/2\right) \qquad p =: \sqrt{\frac{m\hbar\omega(t)}{2}}\left[a^+(t) - a(t)\right]$$

$$\left[a(t), a^+(t')\right] = \frac{1}{2}\left[\sqrt{\frac{\omega(t)}{\omega(t')}} + \sqrt{\frac{\omega(t')}{\omega(t)}}\right] \rightsquigarrow \left[H(t), H(t')\right] \neq 0$$

· $|\dot\psi\rangle = -\frac{i}{\hbar} H(t)|\psi\rangle$ + falls $[H(t), H(t')] = 0 \rightsquigarrow U = \exp\{-\frac{i}{\hbar}\int^t H(t')dt'\}$

+ $H(t) = H(t+T) \rightsquigarrow U(t) = \underbrace{K(t)}_{K(t+T)} e^{-\frac{i}{\hbar} \ldots}$ "Floquet-Theorie"

+ falls $\frac{\langle m(t)|\dot H|n(t)\rangle}{[E_n(t) - E_m(t)]^2} \ll 1$ ($n \neq m$) $\qquad H(t)|n(t)\rangle = E_n(t)|n(t)\rangle$

$\rightsquigarrow$ "adiabatische Näherung"

$$\rightsquigarrow U_{ad}(t) = \sum_n e^{-i\gamma(t)} |n(t)\rangle\langle n(0)|$$

4.3.1. Unitäre Prozessschritte

$$\dot\rho = -\frac{i}{\hbar}[H(t), \rho] \qquad \rho(0) = \sum_n P_n^0 |n(0)\rangle\langle n(0)|$$

$$\rightsquigarrow \rho(t) = \sum_n P_n^0 |n(t)\rangle\langle n(t)| = U_{ad}(t)\rho(0)U_{ad}^+(t)$$

WS bleiben gleich $\rightsquigarrow$ auch die Entropie bleibt gleich

+ von Neumann Entropie

$$S(\rho) = S(U\rho U^+) \qquad \text{immer}: \quad \frac{d}{dt} S = -\mathrm{Tr}\{\dot\rho \ln\rho + \cancel{\rho\rho^{-1}}\cdot\dot\rho\}$$
$$= +\mathrm{Tr}\{\tfrac{i}{\hbar}[H(t), \rho]\ln\rho\} = 0$$

4.3.2. Dissipative Prozessschritte

Lindblad ME
$$\dot\rho = -\frac{i}{\hbar}[H, \rho]$$
$$+ \Gamma(1+n_B)\left[a\rho a^+ - \tfrac{1}{2}a^+a\rho - \tfrac{1}{2}\rho a^+ a\right]$$
$$+ \Gamma n_B\left[a^+\rho a - \tfrac{1}{2}aa^+\rho - \tfrac{1}{2}\rho a a^+\right] = \mathcal{L}\rho$$

$\dot S \neq 0$

Kopplungsstärke Reservoir

$$\frac{1}{e^{\beta\omega} - 1}$$

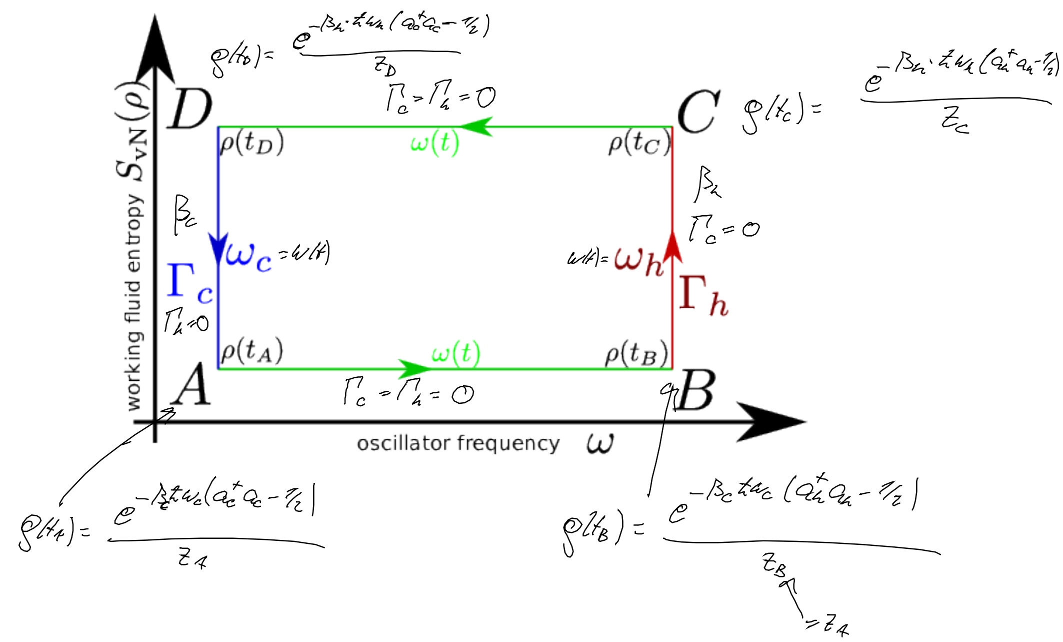
man zeigt: $\mathcal{L}\, e^{-\beta H} = 0$

$$\rho_c = \frac{e^{-\beta H}}{\mathrm{Tr}\{e^{-\beta H}\}} = \sum_n P_n \cdot |n\rangle\langle n| \qquad P_n = \frac{e^{-\beta E_n}}{\sum_m e^{-\beta E_m}}$$

EZ von H

$$\frac{d}{dt}\langle n|\rho|n\rangle = \Gamma(1+n_B)\cdot(n+1)\,\langle n+1|\rho|n+1\rangle$$
$$+ \Gamma n_B \cdot n\,\langle n-1|\rho|n-1\rangle + [\ldots]\underbrace{\langle n|\rho|n\rangle}_{P_n}$$

4.3.3. Otto-Zyklus



<u>Arbeit</u>: $\Delta W_{AB} = \mathrm{Tr}\left\{\hbar\omega_h\left(a_h^\dagger a_h + 1/2\right)\cdot \varrho(t_B)\right\} - \mathrm{Tr}\left\{\hbar\omega_c\left(a_c^\dagger a_c + 1/2\right)\cdot \varrho(t_A)\right\}$

$$= \frac{\hbar\omega_h - \hbar\omega_c}{e^{\beta_c\hbar\omega_c} - 1} + \frac{\hbar(\omega_h - \omega_c)}{2}$$

$$\Delta W_{BC} = \ldots = \frac{\hbar\omega_c - \hbar\omega_h}{e^{\beta_h\cdot\hbar\omega_h} - 1} + \frac{\hbar(\omega_c - \omega_h)}{2}$$

<u>Wärme</u> $\Delta Q_{BC} = \mathrm{Tr}\left\{\hbar\omega_h\left(a_h^\dagger a_h + \tfrac{1}{2}\right)\left[\varrho(t_C) - \varrho(t_B)\right]\right\} = \frac{\hbar\omega_h}{e^{\beta_h\hbar\omega_h} - 1} - \frac{\hbar\omega_h}{e^{\beta_c\hbar\omega_c} - 1}$

ΔQ_{DA} analog

$$\rightsquigarrow \Delta W_{ges} = -\Delta W_{AB} - \Delta W_{CD} = \hbar(\omega_h - \omega_c)\left[\frac{1}{e^{\beta_h\hbar\omega_h} - 1} - \frac{1}{e^{\beta_c\hbar\omega_c} - 1}\right]$$

$$\eta = \frac{\Delta W_{ges}}{\Delta Q_{BC}}\,\Theta(\Delta W_{ges}) = \left(1 - \frac{\omega_c}{\omega_h}\right)\cdot\Theta(\Delta W_{ges})$$

$\beta_h \cdot \omega_h \le \beta_c \cdot \omega_c$

$\omega_h^{max} = \frac{\beta_c}{\beta_h}\cdot\omega_c$

$$\eta_{max} = 1 - \frac{\beta_h}{\beta_c} = 1 - \frac{T_c}{T_h} = \underline{\underline{\eta_{Carnot}}}$$

realistischer: Effizienz bei ΔW_{ges}^{max}

$$\eta_{max}^{CA} = 1 - \sqrt{\frac{T_c}{T_h}} < \eta_{Carnot}$$

noch realistischer: numerisch lösen

Lindblad $\underbrace{\frac{d}{dt} - \mathrm{Tr}\{\varrho \ln \varrho\}}_{\frac{d}{dt} S_{sys}} \underbrace{- \beta\,\mathrm{Tr}\{H\,\dot{\varrho}\}}_{+\frac{d}{dt} S_{res}} \geq 0$

Viel Erfolg bei der Klausur ☺