

Wdh. Übergang in TD-Lines $N \rightarrow \infty$ $V \rightarrow \infty$ $\left. \vphantom{N, V} \right\} \frac{N}{V} = \text{const}$

+ im 3D + gebundene Zust. $\left. \vphantom{\text{im 3D}} \right\} \frac{1}{V} \sum \frac{1}{k} \frac{1}{e^{\beta(\hbar^2 k^2 / 2m)} \pm 1} \xrightarrow{\text{TD-Lines}} \frac{N_{\text{stat}}}{z^2} \int_0^\infty \frac{x^2 dx}{e^{\beta(\hbar^2 x^2 / 2m)} \pm 1}$

o. Fermiideal $z = e^{\beta \mu}$



\Rightarrow Fermi ($T=0$) bleibt endlich

- + Kollaps von Spinnen
- + hat Einfluss auf versch. Eigenschaften von FK
- + resultiert nicht aus Coulomb-Wechselwirkung von e^- in FK, sondern aus Fermi-Statistik

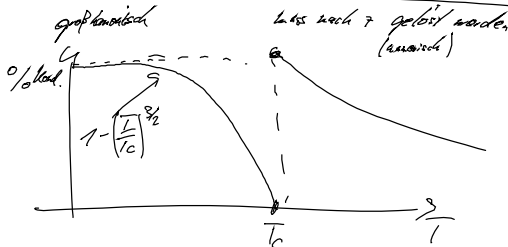
b.) Fermiideal: GZ abschätzen

$$\frac{N}{V} = \frac{z}{V(1-z)} + \left(\frac{4}{2\pi\hbar^3} \right)^{3/2} g_{3/2}(z)$$

Kontinuum-Anteil Anteil in angeregten ZS

$$g_{\lambda}^{\pm}(z) = \frac{1}{\Gamma(\lambda)} \int_0^\infty \frac{x^{\lambda-1}}{z^{-1} e^x \pm 1} dx$$

$$g_{\lambda}(z=1) = \zeta(\lambda)$$



3.2.6 Planck'sches Strahlungsgesetz

Strahlung: $\epsilon(\nu) = \frac{h^3 \nu^3}{2\pi^2} p_{\nu}(\nu)$ $p_{\nu}(\nu)$ ist Boltzmann

also: $\epsilon^2 = p^2 c^2 + h^2 \nu^2 c^4$

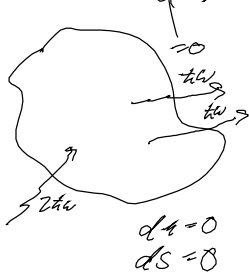
Planckengesetz: $h\nu = 0$ $\epsilon(\nu) = h\nu = \epsilon_0 \nu$

$\epsilon \propto V^{-1/3} \rightarrow \frac{\partial \epsilon}{\partial V} = -\frac{1}{3} \frac{\epsilon}{V}$

$\Rightarrow P_{\text{ir}} = \frac{2}{3} \frac{u}{V}$

$P_{\text{refl}} = \frac{1}{3} \frac{u}{V}$

$dH = TdS - pdV + \mu dN$



$\rightarrow 0 = \mu \cdot dN$
 $dH \neq 0$

$\mu_{\text{stat}} = 0$

$$h\nu = \frac{1}{e^{\beta \hbar \omega} - 1}$$

$$u = \frac{N}{V} = \frac{1}{V} \sum_{\vec{k}} \frac{1}{e^{\beta \hbar c k} - 1} \rightarrow \frac{1}{V} \int \frac{4\pi k^2}{(2\pi)^3} \frac{1}{e^{\beta \hbar c k} - 1} dk \quad N_{ph} = 2$$

$$\frac{N}{V} = \frac{2}{\pi^2} \int_0^\infty \frac{g(\vec{k})}{(\beta \hbar c k)^3} dk \quad \text{Teilchenanzahl}$$

$$\frac{u}{V} = \frac{1}{V} \sum_{\vec{k}} \frac{\hbar c k}{e^{\beta \hbar c k} - 1} \rightarrow \frac{2}{\pi^2} \int_0^\infty \frac{\hbar c k^3}{e^{\beta \hbar c k} - 1} dk = \frac{\pi^2}{15} \frac{1}{(\hbar c)^3 \beta^4} = u = \frac{u}{V}$$

Energieerwartung

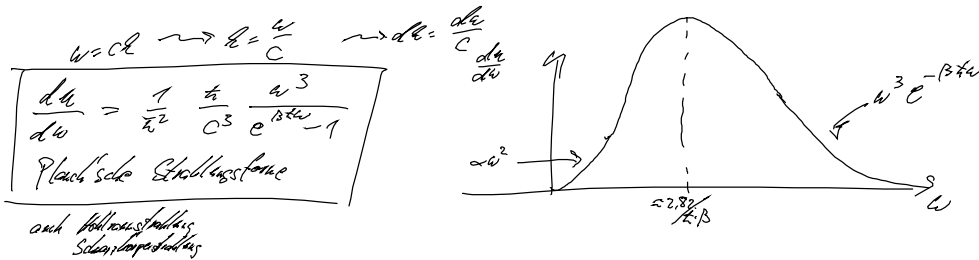
$$P_{rel} = \frac{1}{3} \frac{u}{V} = 0$$

aus Komp.-relation

$$\frac{S}{V} = \frac{1}{T} \left(\frac{u}{V} + P - \frac{N}{V} \right) = \frac{1}{T} \frac{4}{3} \frac{u}{V}$$

$\frac{1}{3} \frac{u}{V}$

$S(T \rightarrow 0) \rightarrow 0$ Nernst'sches WT



3.3 Quanten-Spin-Modelle

Spin (Klassik): $\vec{S}_i \in \{-1, +1\}$
 quant. Operatoren: versch. Anzahle u. kommut. Operatoren korrekten nicht
 falls $[\mu_i, \mu_j] = 0 \rightarrow$ klassische Spin-Systeme

$H = H_0 + H_1 \quad [H_0, H_1] = 0$

3.3.1. Quanten-Ising-Modell

$$\hat{H} = -J \sum_{\langle ij \rangle} \hat{S}_i^x \hat{S}_j^x - \gamma \sum_{i=1}^N \hat{S}_i^z \quad \hat{S}_{i\alpha}^2 = \hat{S}_i^2$$

externes Feld $\gamma > 0$: ferromagnet. NN- u. NW

Klassische Limes $\hat{S}_i^x \rightarrow \hat{S}_i^z$

$$\hat{S}^z |0\rangle = +1 |0\rangle \quad \hat{S}^z |z\rangle = (-1)^z |z\rangle$$

$$\hat{S}^z |1\rangle = -1 |1\rangle \quad z \in \{0, 1\}$$

$$\hat{S}^z = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad [\hat{S}_i^x, \hat{S}_j^y] = 2i \hat{S}_i^z \delta_{ij}$$

$$\hat{S}^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \rightarrow \{ |0\rangle, |1\rangle \}$$

$\alpha \in \{z_1, z_2\}$
 $\hat{S}^z \rightarrow \hat{S}_i^z \quad i \pm j = N$

- 0-temper. Modell welches für $N \rightarrow \infty$ gelöst werden kann
- bricht: nur klassische PT's: unkontroll. Verhalten bei $T = T_c$
- Ising-Modell hat QPT: auch bei $T=0$ bei $g = g_{crit}$

$$\hat{S}_i^z |z_1 \dots z_N\rangle = (-1)^{z_i} |z_1 \dots z_N\rangle \quad \hat{S}_i^x |z_1 \dots z_N\rangle = |z_1 \dots (1-z_i) \dots z_N\rangle$$

a) nur ext. Feld

$$\hat{b}^x | \rightarrow \rangle = \frac{1}{\sqrt{2}} (| \rightarrow \rangle + | \leftarrow \rangle) \rightarrow | \rightarrow \rangle = \frac{1}{\sqrt{2}} (| 0 \rangle + | 1 \rangle)$$

$$| \leftarrow \rangle = \frac{1}{\sqrt{2}} (| 0 \rangle - | 1 \rangle)$$

$| \psi_0(z=0) \rangle = | \rightarrow \rangle \otimes \dots \otimes | \rightarrow \rangle$ "paranagelichte Phase"
 ist gerade: $\sum^x | \psi_0(z=0) \rangle = +1 \cdot | \psi_0(z=0) \rangle$

b) nur HH-WW

für $z \gg \xi$

$$| \psi_0^{(+)}(z=0) \rangle = | 0 \dots 0 \rangle$$

$$| \psi_0^{(-)}(z=0) \rangle = | 1 \dots 1 \rangle$$

"ferromagnetische Phase"

$$\sum^x = \frac{N}{2} \cdot \hat{b}_e^x$$

Billig-Parabel

$$[\sum^x, H] = [\sum^x, \sum^z \hat{b}_i^z; \hat{b}_i^x] (f(z)) = 0$$

$$| \psi_0^{\pm} \rangle = \frac{1}{\sqrt{2}} (| 0 \dots 0 \rangle \pm | 1 \dots 1 \rangle)$$

o Schritte zur Lösung

$$\begin{cases} \hat{D}_n^x = 1 - 2\hat{c}_n^x \hat{c}_{n+1}^x \\ \hat{D}_n^z = -(\hat{c}_n^z + \hat{c}_n^z) \prod_{k=1}^n (1 - 2\hat{c}_k^z \hat{c}_k^z) \end{cases}$$

Fordun-Lessner-Trick

b) $\hat{c}_n = \sum_k \hat{C}_k e^{ikn}$: FT für Leiterop.

↑
 neue Leiterop.

c) $\hat{C}_k = \hat{a}_k \hat{r}_k + \hat{a}_k^x \hat{r}_k^x$ Bogoliubov-Trick

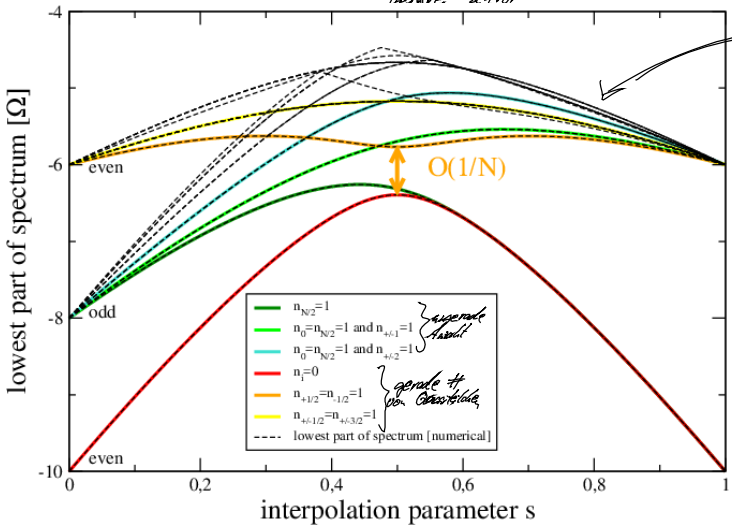
keine Fermionen

$$\epsilon_k = 2\sqrt{q^2 + \gamma^2} - 2\gamma \cdot \gamma \cos\left(\frac{2kx}{\pi}\right)$$

→ am Ende: $H = \hat{P}^- H^+ \hat{P}^- + \hat{P}^+ H^- \hat{P}^+$

$$H^+ = \sum_{k \in \text{BZ}} \epsilon_k (\hat{r}_k^+ \hat{r}_k - \frac{1}{2})$$

normale Teilchen

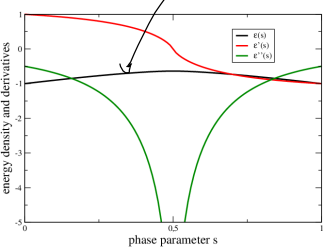


$Z^{10} = 10^{24}$ Basisvekt.

$$E_0 = \sum_k \epsilon_k (-1/2)$$

$$\frac{E_0}{N} = \int_{-1/2}^{+1/2} \epsilon(k) dk$$

$\epsilon(k) =$ Energieerhalte des GZ



$$\begin{aligned} c_2 s &= 9 \\ c_2 (1-s) &= 7 \end{aligned} \quad \text{Sart} = \frac{1}{2} \rightarrow \text{für } 7$$