→ <u>Transport → electrical:</u>

j ... e^- semi-classical

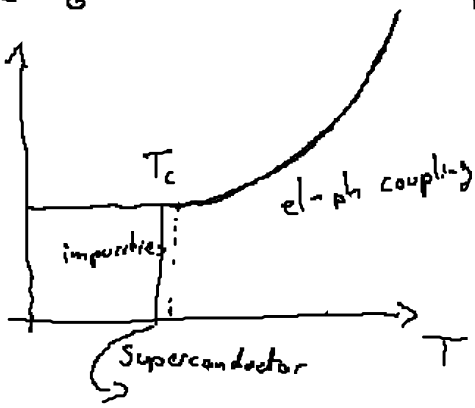
↳ charge

velocity ... $v_g = \frac{1}{\hbar} \frac{\partial \varepsilon_n(\vec{k})}{\partial \vec{k}}$

τ ... finite lifetime ⇐ el-phonon scattering

$\rho = \frac{1}{\sigma}$ ⇐ impurities

<u>Metals</u>



$\rho = \rho_0 + [A_0 + B_0 T^{\alpha}]$

$\alpha \sim 5$

$T < T_c$ → $\rho = 0$ / $\sigma = \infty$

$T_c \sim$ 0 .. K ... 20 K

1986 ... "Unconventional" Superconductor ... Cuprates

⇒ high $T_c \sim 150$ K

→ Mechanism unknown

- Breakdown of the Born-Oppenheimer Approximation
- "Independent" electrons:

$\sigma = \infty$ → currents run ∞ time

↳ 100,000 years ...

Meissner - Ochsenfeld: 1933

"perfect" diamagnet

$$B = \mu_0 \mu H = \mu_0 (1 + \chi) H = 0$$

$\chi = -1$

$\Rightarrow$ The field expelled from the superconductor

$\neq \sigma = \infty$

$\frac{n_s}{n}$ .... conduct $\Rightarrow m \frac{dv_s}{dt} = -eE$

$$\Rightarrow \frac{dj}{dt} = \frac{n_s e^2}{m} E \Rightarrow E = \frac{m}{n_s e^2} \frac{dj}{dt}$$

$$\nabla \times E = -\frac{1}{c} \frac{\partial B}{\partial t}$$

$$\Rightarrow \frac{\partial}{\partial t} \underbrace{\left(\nabla \times j + \frac{n_s e^2}{m} B \right)} = 0$$

classical effect

$\Rightarrow$ eddy currents screen the field

$\Rightarrow$ diamagnetism

$\Rightarrow$ Superconductors:

$$\nabla \times j + \frac{n_s e^2}{m} B \overset{!}{=} 0$$

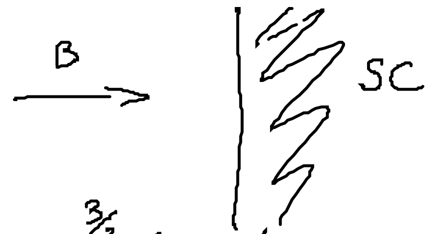
$$\Rightarrow \nabla \times B = \frac{4\pi}{c} j$$

$$\Rightarrow \underbrace{\nabla \times (\nabla \times B)} + \frac{4\pi n_s e^2}{m c^2} B = 0$$

$$\nabla(\nabla B) - \nabla^2 B \qquad \nabla B = 0$$

$$\nabla^2 B = \frac{4\pi n_s e^2}{m c^2} B \Rightarrow$$

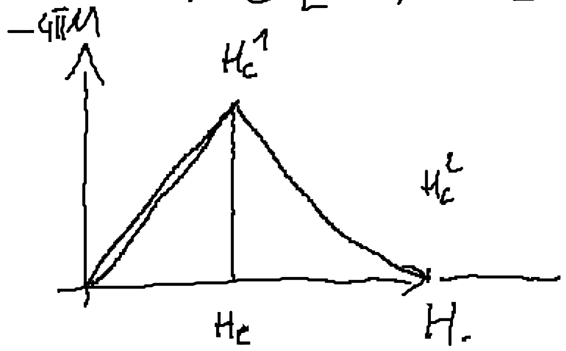
semi-infinite superconductor

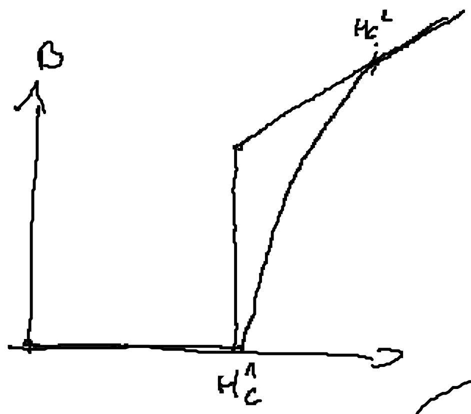


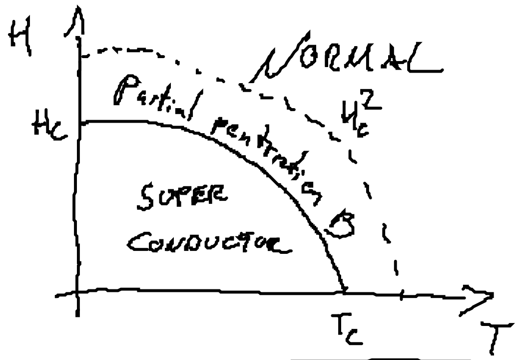
$$\Rightarrow B(x) = B_0 e^{-\frac{x}{\lambda_L}}$$

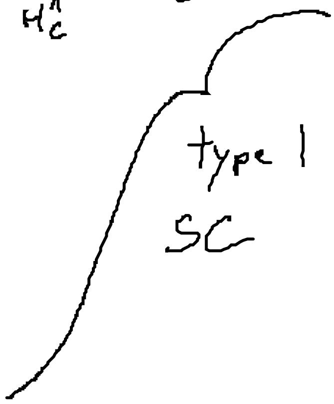
$$\lambda_L = \sqrt{\frac{m c^2}{4\pi n_s e^2}} \approx 42 \cdot \left(\frac{r_s}{a_0}\right)^{3/2} \left(\frac{n}{n_s}\right)^{1/2} \quad [\mathring{A}]$$

$$\Rightarrow \lambda \in [100, 300]\ \mathring{A}$$









H ... superconductor: phase transition becomes first order

→ Anomaleous Heat capacity:

$$C_V \sim T \Rightarrow C_V^{SC} \sim \exp(T)$$

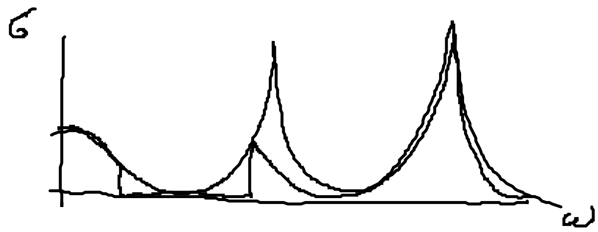
optical phonons, Einstein model

$\Longrightarrow$ "gap" $\sim \Delta$: Misleading term $\neq$ Band gap electronic

$\Rightarrow$ heat conductivity ≈ 0 } j_s carries no entropy
Seebeck effect ≈ 0 }

$\rightarrow$ sound attenenuation

$\rightarrow$ optical conductivies $\rightarrow$ look completely normal, but have a gap (Δ) infrared



Microscopic London equation:

$$\underbrace{j(r) = \frac{ie}{2m}\left(\Psi^{*}\nabla\Psi - \Psi\nabla\Psi^{*}\right) - \frac{e^2}{m} A \Psi^{*}\Psi^{*}}_{\text{QM 1: continuity equation for } \Psi}$$

$\Longrightarrow A = A(r)$ vector potential $\nabla \times A = B$, $\nabla A = 0$ (Maxwell gauge)

$$\Longrightarrow \underline{\underline{SE}}: \left(-\frac{1}{2m}\nabla^2 - \frac{ie}{m}A\nabla + \frac{e}{2m}\right)\phi(r) = E \quad \phi(r)$$

$\Longrightarrow \phi(r)$ ... Boson .. Bose-Einstein-Condensate

$\phi(r) = \phi_o(r) + \phi_e(r)$ (Perturbation Theory)

$\hookrightarrow \phi_o(r) = \frac{1}{\sqrt{V}}$

linearize in A

$$\Rightarrow -\frac{1}{2m}\nabla^2\phi_0 - \frac{1}{2m}\nabla^2\phi_e - \frac{ie}{m}A\nabla\phi_0 = E_0\phi_e + E_1\phi_0$$

scalar

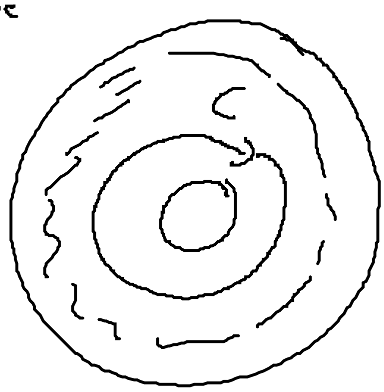
$\Rightarrow \sim \nabla A$

$$\Rightarrow j = -\frac{e^2 n_s}{2m} A \qquad \nabla\times A = B \qquad \nabla\times B = \frac{4\pi}{c} j$$

$$\Longrightarrow \underline{B + \lambda_L \nabla\times B = 0}$$ London equation

$\rightarrow$ Flux quantized:

$$\phi(r) = \sqrt{n_s}\, e^{-\varphi(r)}$$ $\leftarrow$ Phase



$$\phi_B = \int_{surf} ds\, B$$

$$= \int_{surf} ds\, \nabla\times A$$

$$\overset{\text{Stokes}}{=} \oint dl\, A(r)$$

$$0 = j(r) = \frac{ie}{2m}\left(\psi^*\nabla\psi - \psi\nabla\psi^*\right) - \frac{e^2}{2m}A\psi^*\psi$$

$$\Rightarrow -\frac{e^2}{2m}\Big(\underbrace{i\nabla S + i\nabla S}\Big)\phi\phi^* - \frac{e^2}{2m} A\,\phi^*\phi = 0$$

$$\Rightarrow A(r) = -\frac{\nabla S}{e}$$

$$\Rightarrow \phi_B = -\oint_C dl\,\frac{\nabla S}{e} = \frac{\delta S}{e} = \frac{2\pi\hbar c}{e}\,p$$

$$p \in \mathbb{N}$$

$\Rightarrow$ 1961 : Deaver, Fairbank : $e \rightarrow 2e$

exp. hint of Cooper/Ogg pairs

$\Rightarrow$ 1941 : Ogg suggested that $2e^- \Rightarrow 1$ boson $\Big\} \Rightarrow T_c \sim 10^4\,K$

Microscopic Theory : BCS

Bardeen – Cooper – Schriefer

Cooper: requires attractive interaction

$$\underset{\frac{1}{r}}{\bullet \longleftrightarrow \bullet}$$

$\Rightarrow$ Quantum statistics (Fermi)

$\Rightarrow V > 0 \Rightarrow$ Cooper pairs

$\Rightarrow$ Fermi-statistics / Pauli-principle

- El-ph. coupling:

  Make actual measurments of T for different isotopes

  $\Rightarrow T_c(M^{\frac{1}{2}}) \Rightarrow$ phonons