

→ Transport → electrical:

$j \dots e^-$ semi-classical

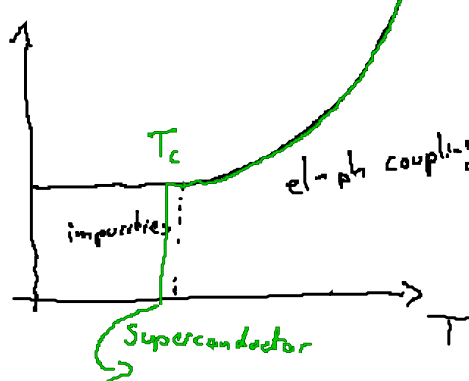
↳ charge
velocity ... $v_g = \frac{1}{\hbar} \frac{\partial \epsilon_n(\vec{k})}{\partial \vec{k}}$

$\tau \dots$ finite lifetime \Leftarrow el-phonon scattering

$S = \frac{1}{G}$

← impurities

Metals



$$S = S_0 + [A_0 + B_0 T^\alpha]$$

$\alpha \sim 5$

$T < T_c \rightarrow S = 0 / G = \infty$

$T_c \sim C \dots K \dots 20K$

1986 ... "Unconventional" Superconductor ... Cuprates

\Rightarrow high $T_c \sim 150K$

→ Mechanism unknown

- Break down of the Born-Oppenheimer Approximation
- "Independent" electrons:

$G = \infty \rightarrow$ currents run ∞ time
↳ 100,000 years ...

Meissner - Ohsenfeld: 1933

"perfect" diamagnet

$$B = \mu_0 \mu H = \mu_0 (1 + \chi) H = 0$$

$\chi = -1$

\Rightarrow The field expelled from the superconductor

$$\neq \sigma = \infty$$

$$\frac{n_s}{n} \dots \text{conduct} \Rightarrow m \frac{dv_s}{dt} = -eE$$

$$\Rightarrow \frac{dj}{dt} = \frac{n_s e^2}{m} E \Rightarrow E = \frac{m}{n_s e^2} \frac{dj}{dt}$$

$$\nabla \times E = -\frac{1}{c} \frac{\partial B}{\partial t}$$

$$\Rightarrow \frac{\partial}{\partial t} \left(\nabla \times j + \frac{n_s e^2}{m} B \right) = 0$$

classical effect
 \Rightarrow eddy currents
screen the field
 \Rightarrow diamagnetism

\Rightarrow Superconductors:

$$\nabla \times j + \frac{n_s e^2}{m} B \stackrel{!}{=} 0$$

$$\Rightarrow \nabla \times B = \frac{4\pi}{c} j$$

$$\Rightarrow \nabla \times (\nabla \times B) + \frac{4\pi n_s e^2}{m c^2} B = 0$$

$$\nabla(\nabla \cdot B) - \nabla^2 B \quad \nabla B = 0$$

semi-infinite superconductor

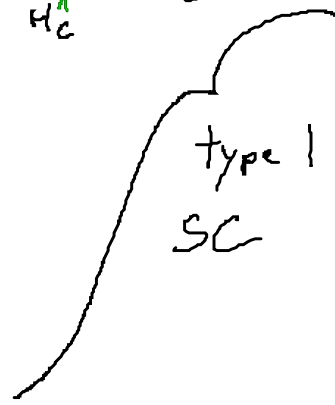
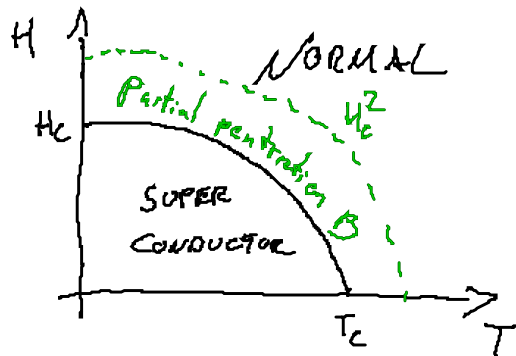
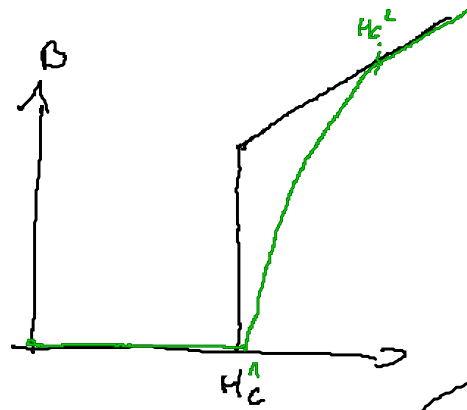
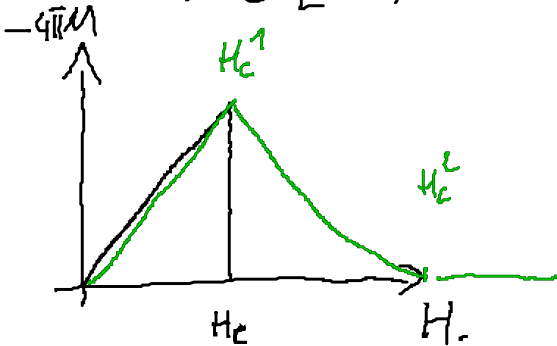
$$\nabla^2 B = \frac{4\pi n_s e^2}{m c^2} B \Rightarrow$$

$$\Rightarrow B(x) = B_0 e^{-\frac{x}{\lambda_L}}$$



$$\lambda_L = \sqrt{\frac{m c^2}{4\pi n_s e^2}} \approx 42 \cdot \left(\frac{r_s}{a_0}\right)^{3/2} \left(\frac{n}{n_s}\right)^{1/2} [\text{\AA}]$$

$$\Rightarrow \lambda \in [100, 300] \text{\AA}$$



$H \dots$ superconductor: phase transition becomes first order

→ Anomalous heat capacity:

$$C_V \sim T \Rightarrow C_V^{SC} \sim \exp(T)$$

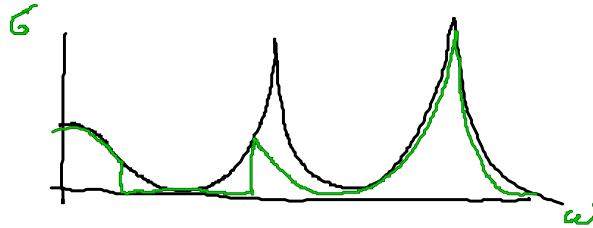
optical phonons, Einstein model

\Rightarrow "gap" - Δ : Misleading term \neq Band gap electronic

\Rightarrow heat conductivity ≈ 0
Seebeck effect ≈ 0 } j_s carries no entropy

\rightarrow sound attenuation

\rightarrow optical conductivities \rightarrow look completely normal,
but have a gap (Δ) infrared



Microscopic London equation:

$$\mathbf{j}(\mathbf{r}) = \frac{ie}{2m} (\psi^* \nabla \psi - \psi \nabla \psi^*) - \frac{e^2}{m} A \psi^* \psi$$

QM1: continuity equation for ψ

$\Rightarrow A = A(\mathbf{r})$ vector potential $\nabla \times A = B$, $\nabla A = 0$ (Maxwell gauge)

$$\Rightarrow \underline{\underline{SE}}: \left(-\frac{1}{2m} \nabla^2 - \frac{ie}{m} A \nabla + \frac{e}{2m} \right) \phi(\mathbf{r}) = E \cdot \phi(\mathbf{r})$$

$\Rightarrow \phi(\mathbf{r})$... Boson ... Bose-Einstein-Condensate

$$\phi(\mathbf{r}) = \phi_0(\mathbf{r}) + \phi_2(\mathbf{r}) \quad (\text{Perturbation Theory})$$

$$\phi_0(\mathbf{r}) = \frac{1}{\sqrt{V}} \quad \text{linearize in } A$$

$$\Rightarrow -\frac{1}{2m} \nabla^2 \phi_0 - \frac{1}{2m} \nabla^2 \phi_e - \frac{ie}{m} A \nabla \phi_0 = E_0 \phi_e + E_1 \phi_0$$

$$\Rightarrow \vec{j} = -\frac{e^2 n_s}{2m} A$$

scalar
 $\Rightarrow \sim \nabla A$

$$\nabla \times A = B \quad \nabla \times D = \frac{4\pi}{c} \vec{j}$$

$$\Rightarrow \underline{B + \lambda_L \nabla \times B = 0} \quad \text{London equation}$$

Flux quantized:

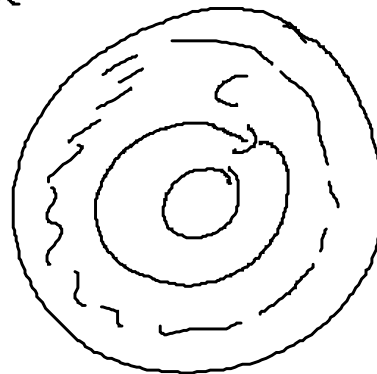
$$\phi(r) = \sqrt{n_s} e^{-i\theta(r)} \quad \leftarrow \text{Phase}$$

$$\phi_0 = \int_{\text{surf}} ds B$$

$$= \int_{\text{surf}} ds \nabla \times A$$

$$\stackrel{\text{Stokes}}{=} \oint dl A(r)$$

$$0 = j(r) = \frac{ie}{2m} (\psi^* \nabla \psi - \psi \nabla \psi^*) - \frac{e^2}{2m} A \psi^* \psi$$



$$\Rightarrow -\frac{e^2}{2m} (i \nabla \mathcal{S} + i \nabla \mathcal{S}) \psi \psi^* - \frac{e^2}{2m} A \psi^* \psi = 0$$

$$\Rightarrow A(r) = -\frac{\nabla \mathcal{S}}{e}$$

$$\Rightarrow \phi_B = - \int_C dl \frac{\nabla \mathcal{S}}{e} = \frac{\mathcal{S}\mathcal{S}}{e} = \frac{2\pi \hbar c}{e} p$$

↓
p ∈ N

⇒ 1961: Deaver, Fairbank: $e \rightarrow 2e$
 exp. hint of Cooper/Ogg pairs

⇒ 1941: Ogg suggested that
 $2e^- \Rightarrow 1 \text{ boson}$ } ⇒ $T_c \sim 10^4 \text{ K}$

Microscopic Theory: BCS

Bardeen - Cooper - Schrieffer

Cooper: requires attractive interaction

$$\leftarrow \frac{1}{v} \rightarrow$$

⇒ Quantum statistics (Fermi)

⇒ $V > 0 \Rightarrow$ Cooper pairs

⇒ Fermi-statistics / Pauli-principle

• El-ph. coupling:

Make actual measurements of T for different isotopes

$$\Rightarrow T_c(M^{\frac{1}{2}}) \Rightarrow \underline{\text{phonons}}$$