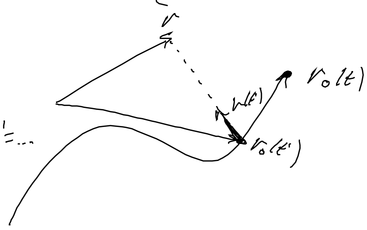
**WdH** ∘ Liénard Wiechert Potentiale

$$\underline{\Phi}(\underline{r},t) = \frac{e}{|r - r_0(t')|\left[1 - \frac{1}{c}\cdot \underline{n}(t')\cdot v(t')\right]} \qquad n(t') = \frac{r - r_0(t')}{|r - r_0(t')|}$$

Ort zur ret. Zeit

$$t' = t - \frac{|r - r_0(t')|}{c}$$

$$\underline{A}(\underline{r},t) = \frac{\frac{e}{c}\cdot \underline{v}(t')}{|\underline{r} - \underline{r}_0(t')|\left[1 - \frac{1}{c}\,\underline{n}(t')\cdot \underline{v}(t')\right]}\Bigg|_{t' = \dots}$$




∘ oszillierende Quellen

$$\underline{j}(\underline{r},t) = \mathrm{Re}\ \underline{j}(\underline{r})\, e^{-i\omega t} \qquad \varrho(\underline{r},t) = \mathrm{Re}\ \varrho(r)\, e^{-i\omega t}$$

$\in \mathbb{C}$

$$\underline{A}(\underline{r}) = \frac{1}{c}\int d^3r'\ \frac{\underline{j}(r')\, e^{ik|r-r'|}}{|r - r'|}$$

\+ Nahfeld : $\underline{A}(\underline{r}) = \frac{1}{c}\int d^3r'\ \frac{\underline{j}(r')}{|r - r'|} \qquad e^{ik|r-r'|} = e^{i2\pi|r-r'|/\lambda} \simeq 1$

\+ Fernfeld : $r' < d \ll \lambda \ll r$

char. Ausdehnung

$$\underline{A}(r) = -ik\cdot \underline{P}\,\frac{e^{ikr}}{r} \qquad \text{ausl. Kugelwelle}$$

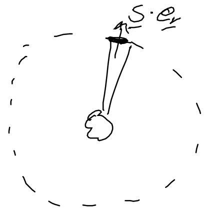
$$\underline{P} = \int \underline{r}\ \varrho(\underline{r})\ d^3r \in \mathbb{C} \qquad \text{stat. Dipolmoment}$$

$\in \mathbb{C}$

$$p(t) = \int \underline{r}\ \varrho(\underline{r},t)\ d^3r = \mathrm{Re}\ \underline{P}\cdot e^{-i\omega t}$$

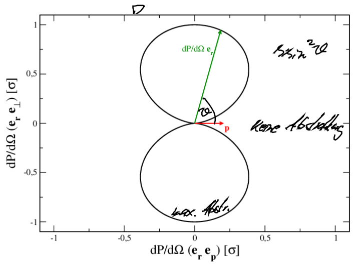
$$\underline{B} = k^2(\underline{e}_r \times \underline{P})\,\frac{e^{ikr}}{r} \qquad \underline{E} = -k^2\,\underline{e}_r \times (\underline{e}_r \times \underline{P})\,\frac{e^{ikr}}{r}$$

\+ Poynting-Vektor $\langle \underline{S}\rangle = \frac{c}{4\pi}\langle \underline{E}(r,t) \times \underline{B}(r,t)\rangle$




$$\frac{dP}{d\Omega} = r^2 \langle \underline{S}\rangle \cdot \underline{e}_r = \frac{\omega^4}{8\pi c^3}\,(\underline{e}_r \times \underline{P})\cdot(\underline{e}_r \times \underline{P}^*)$$

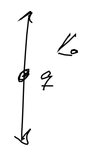
$$\underline{P} = \underbrace{\begin{pmatrix}|P_1|\\ |P_2|\\ |P_3|\end{pmatrix}}_{p} e^{i\delta} \qquad \frac{dP}{d\Omega} = \frac{\omega^4}{8\pi c^3}\,|P_r|^2\cdot \sin^2\vartheta$$



$$P = \int \frac{dP}{d\Omega}\, d\Omega = \frac{\omega^4}{8\pi c^3}\, |p_r|^2 \cdot 2\pi \int_0^{\pi} \sin^2\vartheta \cdot \sin\vartheta\, d\vartheta$$

$$= \frac{\omega^4}{3c^3}\, |p_r|^2 \qquad \text{ges. Strahlungsleistung} \qquad \text{"Dipolformel"}$$

a.) bewegte oszill. PL




$$\rho(\underline{r},t) = q\, \delta(\underline{r} - \underline{r_0} \cdot \cos(\omega t))$$

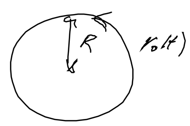
$$\underline{p}(t) = \int \underline{r}\, \rho(\underline{r},t)\, d^3r$$

$$= q \cdot \underline{r_0} \cdot \cos(\omega t) = \mathrm{Re}\, \underbrace{q \cdot \underline{r_0}}_{\underline{P}} \cdot e^{-i\omega t}$$

$$\underline{P} \longrightarrow \underline{P} \in \mathbb{R}^3 \quad (\text{Spezialfall})$$

$$P = \frac{\omega^4 \cdot q^2\, r_0^2}{3\, c^3}$$

b) Kreisbewegung




$$\rho(\underline{r},t) = q \cdot \delta\left(\underline{r} - R \begin{pmatrix} \cos\omega t \\ \sin\omega t \\ 0 \end{pmatrix}\right)$$

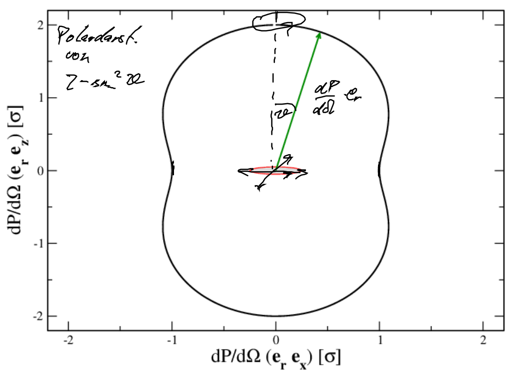
$$\underline{p}(t) = q \cdot R \begin{pmatrix} \cos\omega t \\ \sin\omega t \\ 0 \end{pmatrix} = q \cdot R \cdot \mathrm{Re} \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix} e^{-i\omega t}$$

$$\Rightarrow \underline{P} = q \cdot R \cdot \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix} \qquad \frac{dP}{d\Omega} = \frac{\omega^4}{8\pi c^3} (\underline{e_r} \times \underline{P}) \cdot (\underline{e_r} \times \underline{P}^*)$$

$$|e_r \times P| = q^2 \cdot R^2 \left[2 - \sin^2\vartheta\right]$$

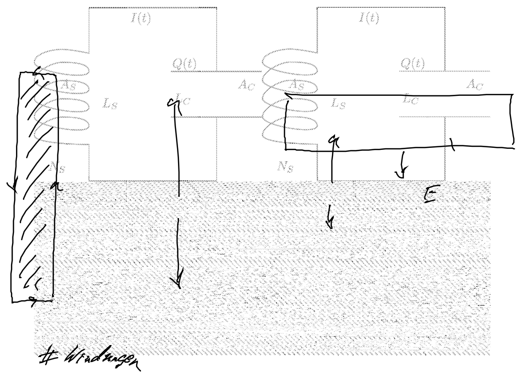
ges. Leistung

$$P = \frac{2}{3} \frac{\omega^4 R^2 \cdot q^2}{c^3}$$




## 6.7. Schwingkreis

Kondensator : Elektrostatik
Spule : Magnetostatik




ES $\quad \nabla \cdot E = 4\pi\rho$

$$\int_V \nabla\cdot E\, d^3r = \oint_{\partial V} \underline{E}\cdot d\underline{A} = 4\pi Q \approx E\cdot A_c$$

$$U = E\cdot l_c = 4\pi Q\cdot \frac{l_c}{A_c} = \frac{1}{C}\cdot Q$$

Kapazität $C = \frac{1}{4\pi}\,\frac{A_c}{l_c}$

**MS**: $\nabla\times B = \frac{4\pi}{c}\cdot j$

$$l_s\cdot B = \frac{4\pi}{c}\, I\cdot N_S$$

magn. Fluss $\Phi_S = A_S\cdot B = $

$$= A_S\cdot \underbrace{\frac{4\pi}{c}\cdot\frac{N_S}{l_S}\cdot I}_{B}$$

$$\Phi_S \sim I$$

$$I = \frac{N_S}{c\cdot L}\cdot \Phi_S \qquad L \hat{=} \text{ Induktivität}$$

$$L = \frac{N_S}{c}\cdot\frac{\Phi_S}{I} = 4\pi\cdot A_S\,\frac{N_S^2}{l_S\cdot c^2}$$

phänomenologisch

einfach Gleichungen zeitabhängig machen

$$Q(t) \approx C\cdot U(t)$$

$$I(t) \approx \frac{N_S}{c\cdot L}\cdot\Phi_S(t)$$

$$\nabla\times\underline{E} + \frac{1}{c}\partial_t\underline{B} = 0$$

$$\underbrace{\oint_{\partial F}\underline{E}\cdot d\underline{r}}_{U} = -\frac{1}{c}\partial_t\underbrace{\iint_F \underline{B}\cdot d\underline{F}}_{\Phi}$$

$$U(t) = -\frac{N_S}{c}\cdot\partial_t\Phi_S \qquad \Phi_S = A_S\cdot B$$

$$\frac{d}{dt}Q(t) = I(t)$$

$$U(t) = -L\cdot\frac{dI}{dt} = -L\cdot\frac{d^2Q}{dt^2} = -L\cdot C\cdot\frac{d^2U}{dt^2}$$

$$\frac{d^2U}{dt^2} + \frac{1}{LC}\cdot U(t) = 0 \qquad \omega = \frac{1}{\sqrt{L\cdot C}} \quad \text{Schwingkreis-Frequenz}$$

analog: $\ddot{Q} + \frac{1}{LC} Q = 0$

mit Widerstand $\ddot{Q} + \frac{R}{L} \cdot \dot{Q} + \frac{1}{LC} \cdot Q = 0$

charakt. Länge

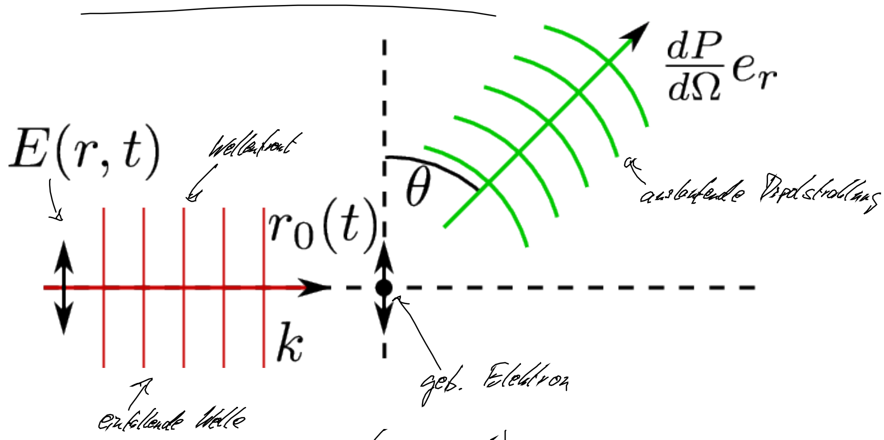
$\nabla \times E = -\frac{1}{c} \partial_t B \approx 0$ (Kondensator)

$\nabla \times B = \frac{1}{c} \partial_t E + \frac{4\pi}{c} j \approx \frac{4\pi}{c} \cdot j$ (Spule)

$\Big\} \quad \frac{\omega \cdot l}{c} \ll 1$

~~falls~~ $\frac{\omega \cdot l}{c} \longrightarrow \frac{1}{ns} \overset{!}{\ll} 1$

## 6.8. Streuung von Licht




$\underline{E}(r,t) = \mathrm{Re}\ \underline{E}_0 \cdot e^{i(\underline{k} \cdot \underline{r} - \omega t)}$ $\qquad$ $\underline{B}(\underline{r},t) = \frac{\underline{k}}{|\underline{k}|} \times \underline{E}(\underline{r},t)$

BWGL für geb. Elektron: nichtrelativistisch

$m_e \cdot \ddot{r}_0(t) + m_e \Gamma \dot{r}_0(t) + m_e \cdot \omega_0^2 \, r_0(t) = -e \, \mathrm{Re}\, \underline{E}_0 \cdot e^{i(\underline{k} \cdot \underline{r}_0 - \omega t)} + \ldots$

Rückstellkraft

Dämpfung: + Stoßprozesse
+ Abstrahlung

Langwellen-Näherung $\qquad \lambda \gg \underline{r}_0(t) \qquad e^{i k \cdot r_0(t)} \approx 1$

$\Rightarrow m_e \cdot \ddot{\underline{r}}_0 + m_e \cdot \Gamma \cdot \dot{\underline{r}}_0 + m_e \cdot \omega_0^2 \, \underline{r}_0 = -e \, \mathrm{Re}\, \underline{E}_0 \cdot e^{-i\omega t}$

Lösung: homog. Lösung + 1 spez. der inhomog. DGL fällt für $t \to \infty$ ab

$\Rightarrow$ für asymptot. Langzeitlimes reicht es, die erzwungene Schw. zu betrachten

Ansatz $r_0(t) = \mathrm{Re}\, \underline{a} \cdot e^{-i\omega t}$

$$\left(-\omega^2 - i\Gamma \cdot \omega + \omega_0^2\right) \cdot \underline{a} = \frac{-e}{m_e} \underline{E}_0 \longrightarrow \underline{a} = \left( \quad \ldots \quad \right) \underline{E}_0$$

$$\underline{P}(t) = -e \cdot r_0(t) = -e \cdot \mathrm{Re} \cdot a \cdot e^{-i\omega t}$$

$$= \mathrm{Re} \frac{e^2/m_e}{\omega_0^2 - \omega^2 - i\Gamma \cdot \omega} E_0 \, e^{-i\omega t}$$

$$\underline{P} = \frac{e^2/m_e}{\omega_0^2 - \omega^2 - i\Gamma \cdot \omega} \underline{E_0} = \alpha_e(\omega) \cdot \underline{E_0}$$

"elektr. Polarisierbarkeit"

$$\frac{dP}{d\Omega} = \frac{c}{8\pi} \left(\frac{e^2}{m_e c^2}\right)^2 \frac{\omega^4}{(\omega^2 - \omega_0^2)^2 + \Gamma^2 \omega^2} |E_0|^2 \cdot \sin^2 \vartheta$$

$\to$ Streuung von Licht an Atomen