

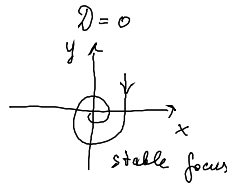
lecture 15 summary

3. Noise-induced dynamics

- systems without ext. periodic forcing
- deterministic system has a stable fixed point

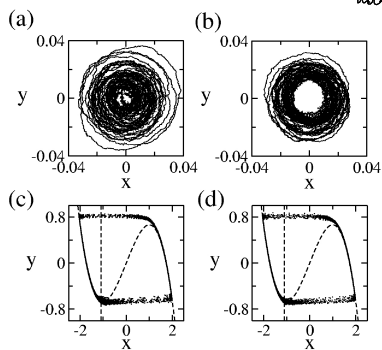
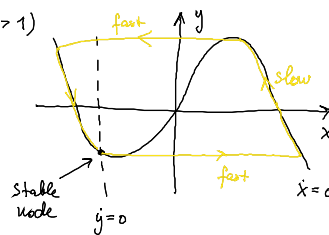
1. Example: van der Pol system
below Hopf bifurcation ($\varepsilon < 0$)

$$\begin{cases} \dot{x} = y \\ \dot{y} = \underbrace{(\varepsilon - x^2)}_{\text{nonlinear damping}} y - \omega_0^2 x + \sqrt{2D} \xi(t) \end{cases}$$



2. Example: Fitz Hugh-Nagumo model
in excitable regime ($a > 1$)

$$\begin{cases} \varepsilon \dot{x} = x - \frac{x^3}{3} - y \\ \dot{y} = x + a + \sqrt{2D} \xi(t) \end{cases}$$



van der Pol system
 $D = 0.003$

Fitz Hugh-Nagumo system
 $D = 0.09$

$D \neq 0$: noise-induced oscillations in FHN model

Janson, Balana, Schöll: PRL 93, 010601 (2004)

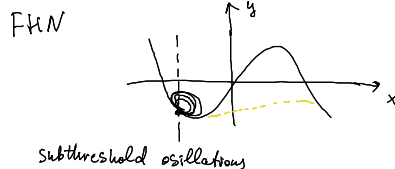
Balana, Janson, Schöll: Physica D 199, 1 (2004)

Schöll, Balana, Janson, Meinan: Stoch. Dyn. 5, 281 (2005)

3.3 Coherence Resonance (CR)

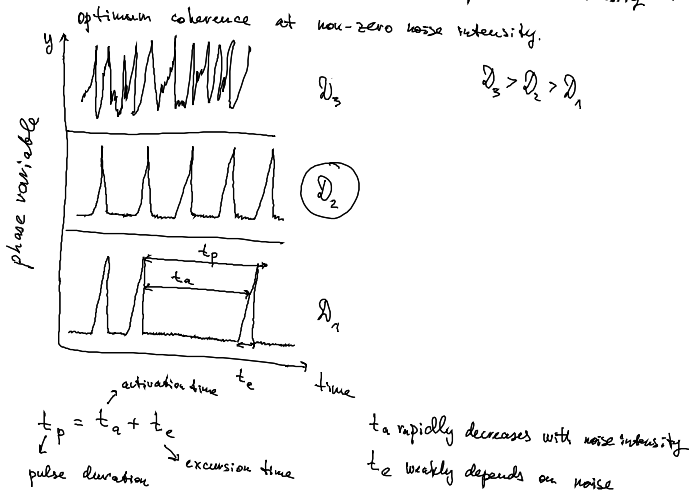
- constructive role of noise

CR: the best regularity (coherence) of noise-induced oscillations is achieved for intermediate/optimal noise intensity D_{opt} .



Subthreshold oscillations

Two competing time scales with opposite dependence upon noise intensity \rightarrow
 non monotonic dependence of coherence upon noise intensity \rightarrow
 optimum coherence at non-zero noise intensity.



Measures of coherence resonance

- normalized standard deviation of interspike interval (ISI)
 (normalized fluctuations of the pulse durations)

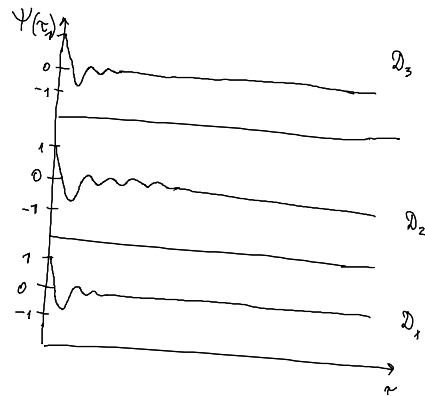
$$R_{IT} = R = \frac{\sqrt{\langle t_p^2 \rangle - \langle t_p \rangle^2}}{\langle t_p \rangle} = \frac{\sqrt{\text{var}(t_p)}}{\langle t_p \rangle}$$

Pikovsky, Kurths, PRL 78, 775 (1997) CR in FHN, the name CR

Gang, Ditzinger, Ming, Haken, PRL 71, 807 (1993)
 "Stochastic resonance without external periodic forcing"

R_{IT} can be used in the case of spiking dynamics
 (not used for non-excitable systems since there are no spikes)

- correlation time
 $t_{cor} = \frac{1}{\Psi(0)} \int_0^{\infty} |\Psi(s)| ds$

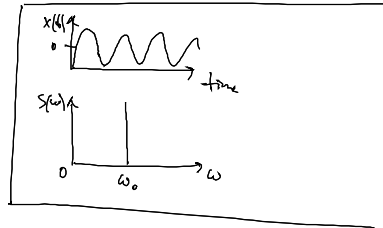
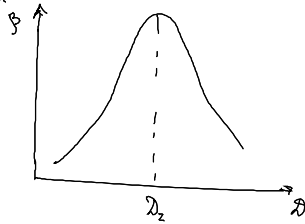
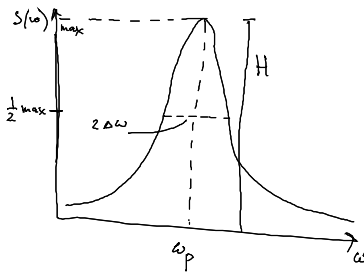


- signal-to-noise ratio (SNR)
of the spectral peak height H and its relative width

$$\beta = \frac{H}{\Delta\omega/\omega_p}$$

HWHM
half width at half maximum $\rightarrow \Delta\omega$

FWHM
full width at half maximum $\rightarrow 2\Delta\omega$



Correlation time

$$t_{cor} = \frac{1}{\Psi(0)} \int_0^{\infty} |\Psi(s)| ds$$

$$\text{ACF: } \Psi(s) = \langle [x(t+s) - \langle x \rangle][x(t) - \langle x \rangle] \rangle$$

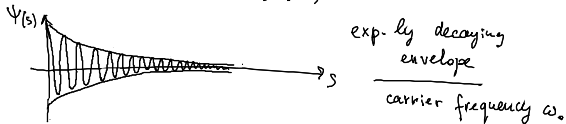
$$\Psi(0) = \sigma^2$$

Motivation of definition

for linear stoch. processes $\dot{x} = -(k+i\omega_0)x + \xi(t)$

(Ornstein-Uhlenbeck proc.
 $\text{Re } e^{-(k+i\omega_0)s}$)

$$\Psi(s) = \Psi(0) e^{-ks} \cos(\omega_0 s)$$



$k > 0$ (stable fixed point)

Relation between k and t_{cor}

$$t_{cor} = \int_0^{\infty} e^{-ks} |\cos \omega_0 s| ds$$

$$\text{approx. } \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} \cos \varphi d\varphi = \frac{2}{\pi} \text{ for } k \ll \omega_0 \text{ (filling factor)}$$

$$\Rightarrow t_{\text{cor}} \approx \frac{2}{\pi} \int_0^{\infty} e^{-ks} ds = \frac{2}{\pi k}$$

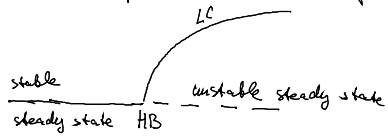
$$\text{therefore } \Psi(s) = \Psi(0) e^{-\frac{2}{\pi} \frac{s}{t_{\text{cor}}}} \cos(\omega_0 s)$$

$$\left. \begin{array}{l} t_{\text{cor}} \approx \frac{2}{\pi k} \\ -k = \frac{-2}{\pi t_{\text{cor}}} \end{array} \right\}$$

$$K = |\text{Re}(\text{eigenvalue of the fixed point})|$$

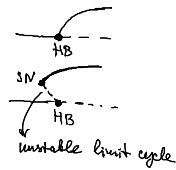
$$\text{Re } \lambda < 0$$

bifurcation parameter (distance from HB)



The more stable is the fixed point, the shorter is correlation time (further away from HB)

Hopf bifurcation \rightarrow supercritical case
 HB \downarrow subcritical case



SN - saddle-node bifurcation of limit cycles

between SN and HB \rightarrow bistability: stable fixed point and stable limit cycle

