WdH + TD der Mastergleichung

$$\bar\varrho_B = \frac{e^{-\beta H_B}}{Z_B} \xrightarrow{KMS} C_{\alpha\beta}(\tau) = C_{\beta\alpha}(-\tau - i\beta) \longrightarrow \bar\varrho_S = \frac{e^{-\beta H_S}}{Z_S}$$

$$\gamma_{\alpha\beta}(-\omega) = \gamma_{\beta\alpha}(+\omega)\, e^{-\beta\omega}$$

$$\mathcal{L}_{BMS}\, \bar\varrho_S = 0$$

→ BMS ist eine Lindblad-Davies Abbildung

falls: $\bar\varrho_B = \frac{e^{-\beta(H_B - \mu N_B)}}{Z_B} \xrightarrow[{[H_I, N_S + N_B] = 0}]{[N_B, H_B]=0 \quad [N_S, H_S]=0} \bar\varrho_S = \frac{e^{-\beta(H_S - \mu N_S)}}{Z_S}$

- von-Neumann Entropie $S(\varrho) = -\mathrm{Tr}\{\varrho \ln \varrho\} = -\sum_n \varrho_n \cdot \ln \varrho_n$

EW von $\varrho$

Bsp: $\varrho = 1 \cdot |\Psi_+\rangle\langle\Psi_+| + 0\, |\Psi_-\rangle\langle\Psi_-|$

$|\Psi_+\rangle = \frac{1}{\sqrt 2}[|0\rangle + |1\rangle]$ $\quad \varrho = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

$|\Psi_-\rangle = \frac{1}{\sqrt 2}[|0\rangle - |1\rangle]$

$\varrho_- = 0$

$\varrho_+ = 1$ $\quad S(\varrho) = 0$

- Spohn'sche Ungleichung $\mathcal{L}\bar\varrho = 0$

$$-\mathrm{Tr}\{(\mathcal{L}\varrho)[\ln\varrho - \ln\bar\varrho]\} \geq 0$$

$$\frac{d}{dt} S(\varrho) + \frac{d}{dt} S_{res} \geq 0 \qquad \text{2. HS}$$

- Coarse-graining $\tilde{\mathcal{U}} = \mathbb{1} - i\int_0^t \tilde H_I(t')dt' - \underbrace{\int_0^t dt_1 \int_0^{t_1} dt_2\, \tilde H_I(t_1)\tilde H_I(t_2)}_{\int_0^t dt_1 \int_0^t dt_2\, \tilde H_I(t_1)\tilde H_I(t_2)\,\Theta(t_1 - t_2)} \pm \ldots$

$$\mathrm{Tr}_B\{\tilde{\mathcal{U}}(t)\, \varrho_S^0 \otimes \bar\varrho_B\, \tilde{\mathcal{U}}^\dagger(t)\} \approx [\mathbb{1} + \mathcal{L}_t \cdot t + \ldots]\varrho_S^0 \qquad \underline{\tau = t}$$

$$\mathcal{L}_\tau \varrho_S^0 = \frac{1}{\tau}\mathrm{Tr}_B\Big\{\int_0^t H_I(t')dt'\, \varrho_S^0 \int_0^t H_I(t')dt' - \int_0^t dt_1 \int_0^{t_1} dt_2\, H_I(t_1) H_I(t_2)\varrho_S^0 - \int_0^t dt_1 \int_0^{t_1} dt_2\, \varrho_S^0 H_I(t_2) H_I(t_1)\Big\}$$

$$\Theta(t_1 - t_2) = \frac{1}{2}(1 + \mathrm{sgn}(t_1 - t_2))$$

=> $\mathcal{L}_\tau$ ist eine Lindblad Form

$\mathcal{L}_\tau$ ist auch gut für kurze Zeiten

$\varrho_S(t) = e^{\mathcal{L}_t \cdot t}\varrho_S^0$ $\qquad e^{\mathcal{L}_\tau \cdot t}\varrho_S^0$ ist immer eine valide DM

"dynamisches coarse-graining"

BMS: 3 Näherungen: + schw. Kopplung, schnell abf. Korrel.-Funktionen + lange Zeiten $\qquad$ DCG: - schwache $(t \cdot H_I)$ - abfallende Korrelationsfunktionen

$$\frac{d}{dt}\rho_S(t) = \underbrace{\left[\left(\frac{d}{dt} e^{\mathcal{Z}_t \cdot t}\right) e^{-\mathcal{Z}_t \cdot t}\right]}_{\mathcal{Z}^{DGB}(t)} \underbrace{e^{+\mathcal{Z}_t \cdot t} \rho_S^0}_{\rho_S(t)}$$

i. A. keine Lindblad Form

$\sum_{ab} |a\rangle\langle a| e^{+iH_S t} A_\alpha e^{-iH_S t} |b\rangle\langle b|$

$[H_B, \rho_B] = 0$

$\rightsquigarrow C_{\alpha\beta}(t_1-t_2) = \frac{1}{2\pi}\int d\omega\, \gamma_{\alpha\beta}(\omega) e^{-i\omega(t_1-t_2)}$

$C_{\alpha\beta}(t_1-t_2)\,\mathrm{sgn}(t_1-t_2) = \frac{1}{2\pi}\int \sigma_{\alpha\beta}(\omega) e^{-i\omega(t_1-t_2)} d\omega$

$A_\alpha^{\omega,\tau} = \int_0^\tau dt\, \tilde{A}_\alpha(t) e^{-i\omega t}$

CG-Mastergleichung ($\tau$ fixiert)

$$\dot{\tilde{\rho}}_S = -i\left[\frac{1}{2i\tau}\frac{1}{2\pi}\sum_{\alpha\beta}\int d\omega\, \sigma_{\alpha\beta}(\omega) A_\alpha^{\omega,\tau} A_\beta^{-\omega,\tau}, \tilde{\rho}_S\right]$$
$$+ \frac{1}{\tau}\sum_{\alpha\beta}\frac{1}{2\pi}\int d\omega\, \gamma_{\alpha\beta}(\omega)\left[A_\beta^{-\omega,\tau}\tilde{\rho}_S A_\alpha^{+\omega,\tau} - \frac{1}{2}\left\{A_\alpha^{+\omega,\tau} A_\beta^{-\omega,\tau}, \tilde{\rho}_S\right\}\right]$$

nutze $\int_0^\tau e^{-i\alpha_\alpha \cdot t} dt = \tau \cdot e^{-i\alpha_\alpha \tau/2} \cdot \mathrm{sinc}\left(\frac{\alpha_\alpha \cdot \tau}{2}\right)$ $\qquad \mathrm{sinc}(x) = \frac{\sin(x)}{x}$

$$\lim_{\tau\to\infty} \tau \cdot \mathrm{sinc}\left[\frac{\tau}{2}(\Omega_a - \omega)\right] \mathrm{sinc}\left[\frac{\tau}{2}(\Omega_b - \omega)\right] = 2\pi \cdot \delta_{\Omega_a,\Omega_b} \cdot \delta(\Omega_a - \omega)$$

$$\boxed{\lim_{\tau\to\infty} \mathcal{Z}_\tau^{CG} = \mathcal{Z}_{BMS}}$$

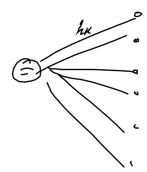
## 1.3.6. Beispiel: Spin-Boson-Modell

$H_S = \Omega \cdot \sigma^z + T\sigma^x$ $\qquad H_B = \sum_k \omega_k b_k^\dagger b_k$

$H_I = \underbrace{\sigma^z}_{A_1} \otimes \underbrace{\sum_k (h_k b_k + h_k^* b_k^\dagger)}_{B_1}$

Verteilung der Kopplungsstärke $\qquad$ Verteilung der Bad-Energien

$\Gamma(\omega) = 2\pi\sum_k |h_k|^2 \delta(\omega - \omega_k)$ $\qquad$ spektrale Dichte




- exakt lösbar für $T=0$ : $[H_S, H_I] = 0$ "pure dephasing"
- $\mathrm{Tr}\{\tilde{B}_1(\tau) \cdot B_1 \bar{\rho}_B\} = \sum_k |h_k|^2 \left[[1+n_B(\omega_k)] \cdot e^{-i\omega_k\tau} + n_B(\omega_k) \cdot e^{+i\omega_k\tau}\right]$

$= C(\tau)$

$$= \frac{1}{2\pi}\int_0^\infty \left[\Gamma(\omega)\,[1+n_B(\omega)]e^{-i\omega\tau} + n_B(\omega) e^{+i\omega\tau}\,\Gamma(\omega)\right] d\omega = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \underbrace{\tilde{\Gamma}(\omega)[1+n_B(\omega)]}_{\gamma(\omega)} e^{-i\omega\tau} d\omega$$

$\tilde{\Gamma}(-|\omega|) = -\Gamma(+|\omega|)$

häufige Annahme: $\tilde{\Gamma}(\omega) = \Gamma_0 \cdot \omega\, e^{-|\omega|/\omega_c}$

ohmsche SD $\qquad$ cutoff

• exakte Lösung $T=0$

$$\tilde{A}_1(t) = e^{+iH_S t}\sigma_z e^{-iH_S t} = \sigma_z \qquad \tilde{B}_1(t) = \sum_k \left(h_k b_k e^{-i\omega_k t} + h_k^* b_k^+ e^{+i\omega_k t}\right)$$

halbe Polaron-Trafo

$$U_P = \exp\left\{-\sigma^z \underbrace{\sum_k \left(\frac{h_k}{\omega_k} b_k - \frac{h_k^*}{\omega_k} b_k^+\right)}_{B}\right\} \qquad U = \cosh(B) - \sinh(B)\cdot\sigma^z \rightsquigarrow U_P^{-1} = U_P^+$$

$$e^A B e^{-A} = \sum_{n=0}^{\infty} \frac{1}{n!}[A,B]_n \qquad [A,B]_0 = B \qquad [A,B]_n = [A,[A,B]_{n-1}]$$

$$\boxed{\begin{aligned} &\bullet\ U_P \sigma^z U_P^+ = \sigma^z \\ &\bullet\ U_P b_k U_P^+ = b_k + \left[+\sigma^z \frac{h_k^*}{\omega_k} b_k^+, b_k\right] \\ &\qquad = b_k - \frac{h_k^*}{\omega_k}\cdot\sigma^z \end{aligned}}$$

• $U_P \sigma^\pm U_P^+ = e^{\pm 2\sum_k \left(\frac{h_k^*}{\omega_k} b_k^+ - \frac{h_k}{\omega_k} b_k\right)} \sigma^\pm$

$$U_P H U_P^+ = \Omega\sigma^z + \sigma^z \sum_k \left(h_k b_k + h_k^* b_k^+ - \frac{|h_k|^2}{\omega_k}\cdot\sigma^z\cdot 2\right) + \sum_k \omega_k \left(b_k^+ - \frac{h_k}{\omega_k}\sigma^z\right)\left(b_k - \frac{h_k^*}{\omega_k}\sigma^z\right)$$

$$= \Omega\sigma^z - \sum_k \frac{|h_k|^2}{\omega_k} + \sum_k \omega_k b_k^+ b_k$$

Spin-Boson-Entkopplung ?

$$\langle \sigma^\alpha \rangle = \mathrm{Tr}\left\{e^{+iHt}\sigma^\alpha e^{-iHt}\rho_0\right\} = \ldots$$

$$\boxed{\tilde{\rho}_{01}(t) = \exp\left\{-\frac{4}{\pi}\int_0^\infty \Gamma(\omega)\frac{\sin^2(\omega t/2)}{\omega^2}\cdot\coth\left(\frac{\beta\omega}{2}\right)d\omega\right\}\rho_{01}(0)}$$

"Dekohärenz"

$$\tilde{\rho}_{00}(t) = \rho_{00}(0) \qquad \tilde{\rho}_{11}(t) = \rho_{11}(0)$$

• jetzt: BMS Mastergleichung ($T \neq 0$)

$$\gamma(\omega) = \Gamma(\omega)[1 + n_B(\omega)] \longrightarrow \sigma(\omega)$$

$$H_S|\pm\rangle = E_\pm|\pm\rangle \qquad E_\pm = \pm\sqrt{\Omega^2+T^2}$$

$$\gamma_{-+,-+} = \Gamma\left(+2\sqrt{\Omega^2+T^2}\right)\left[1 + n_B\left(+2\sqrt{\Omega^2+T^2}\right)\right]|\langle -|\sigma^z|+\rangle|^2 \xrightarrow{T\to 0} 0$$

$$\gamma_{+-,+-} = \Gamma(\quad '' \quad)\, n_B\left(+2\sqrt{\Omega^2+T^2}\right)|\langle -|\sigma^z|+\rangle|^2 \xrightarrow{T\to 0} 0$$

$$\gamma_{--,++} = \gamma(0)\langle -|\sigma^z|-\rangle\langle +|\sigma^z|+\rangle = \gamma_{++,--}$$

$\rightarrow$ in BMS

$$\frac{d}{dt}\rho_{--} = +\gamma_{-+,-+}\rho_{++} - \gamma_{+-,+-}\rho_{--}$$

$$\frac{d}{dt}\rho_{++} = +\gamma_{+-,+-}\rho_{--} - \gamma_{-+,-+}\rho_{++}$$

$$\frac{d}{dt}\rho_{-+} = -i(E_- - E_+ + \sigma_{--} - \sigma_{++})\rho_{-+} + \underbrace{\left(\gamma_{--,++} - \frac{\gamma_{-+,-+} + \gamma_{+-,+-}}{2}\right)}_{\leq 0}\rho_{-+}$$

$$= \eta\cdot\rho_{-+}$$

$$\frac{d}{dt}\begin{pmatrix}\rho_{--}\\\rho_{++}\\\rho_{-+}\\\rho_{+-}\end{pmatrix} = \underbrace{\begin{pmatrix} -\gamma_{+-,+-} & +\gamma_{-+,-+} & 0 & 0 \\ +\gamma_{+-,+-} & -\gamma_{-+,-+} & 0 & 0 \\ 0 & 0 & \eta & 0 \\ 0 & 0 & 0 & \eta^* \end{pmatrix}}\begin{pmatrix}\rho_{--}\\\rho_{++}\\\rho_{-+}\\\rho_{+-}\end{pmatrix}$$

$$= \begin{pmatrix} \mathcal{L}_{pop} & 0 \\ 0 & \mathcal{L}_{coh} \end{pmatrix}$$

$\Rightarrow$ $\bar{\rho}_S$ stimmt mit exakter Lösung überein für $T=0$

$\Rightarrow$ zeitliche Dynamik stimmt nicht ganz

Ausblick $\bar{\rho}_S = \mathrm{Tr}_B\left\{\frac{e^{-\beta H_{tot}}}{Z_{tot}}\right\}$