Wdh.

$\dot{\rho} = \mathcal{L}^{(0)}\rho \hat{=} -i[H_S, \rho]$

- $\mathcal{L} = \mathcal{L}^{(0)} + \sum_\nu \mathcal{L}^{(\nu)}$ (spezifisch für Reserv. ν)

$$\mathcal{L}^{(\nu)} \frac{e^{-\beta_\nu [H_S - \mu_\nu N_S]}}{Z_S} = 0$$

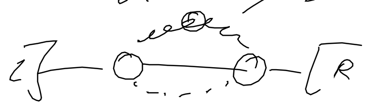
$$\left.\begin{array}{l} I_E^{(\nu)} = \mathrm{Tr}\{H_S (\mathcal{L}^{(\nu)}\rho)\} \\ I_M^{(\nu)} = \mathrm{Tr}\{N_S (\mathcal{L}^{(\nu)}\rho)\} \end{array}\right\} \quad \dot{Q}^{(\nu)} \equiv I_E^{(\nu)} - \mu_\nu I_M^{(\nu)}$$

$$\Rightarrow \dot{S}_i \equiv \dot{S} - \sum_\nu \beta_\nu \dot{Q}^{(\nu)} = \dot{S} + \sum_\nu \dot{S}^{(\nu)}_{res} \geq 0 \qquad \text{2. HS}$$

($\dot{S}$: Änderung der System-Entropie)

Steady-state: $\dot{S} = 0 \rightsquigarrow \dot{S}_i = -\sum_\nu \beta_\nu [I_E^{(\nu)} - \mu_\nu I_M^{(\nu)}] \geq 0$

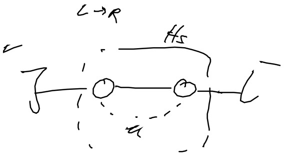
- z. Bsp.: 2-Terminal



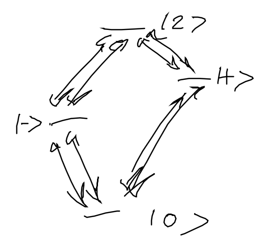
$$\bar{\dot{S}}_i = (\beta_R - \beta_L)\bar{I}_E + (\beta_L\mu_L - \beta_R\mu_R)\cdot\bar{I}_M \geq 0 \longrightarrow \text{Carnot-Beschränkungen}$$

($\bar{I}_E$: $L \to R$)

- Bsp.: SET, DQD



$|0\rangle$, $|-\rangle$, $|+\rangle$, $|2\rangle$



BMS: $\mathcal{L}^{pop}_{BMS}$

$$\rho = \begin{pmatrix} \rho_{0,0} & 0 & 0 & 0 \\ 0 & \rho_{-,-} & \rho_{-+} & 0 \\ 0 & \rho_{+-} & \rho_{+,+} & 0 \\ 0 & 0 & 0 & \rho_{2,2} \end{pmatrix}$$

→ Ratengleichung

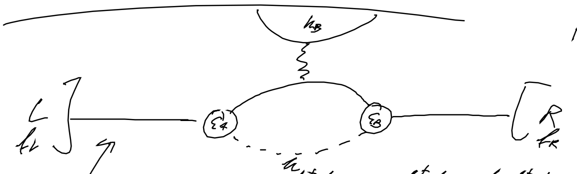
separate Gleichung (Zerfall)

$\dot{\rho}_{-+} = \kappa \cdot \rho_{-+}$ $\quad \mathrm{Re}(\kappa) < 0$

$$\mathcal{L}_{6\times6} = \begin{pmatrix} \mathcal{L}^{pop}_{4\times4} & 0 & 0 \\ -0- & \kappa & 0 \\ -0- & 0 & \kappa^* \end{pmatrix}$$

Blockstruktur gilt nur für nicht entartete Systeme

2.7. Phononen-Assistiertes Tunneln



$$H_S = \varepsilon_A d_A^\dagger d_A + \varepsilon_B d_B^\dagger d_B + U d_A^\dagger d_A d_B^\dagger d_B$$

$$H_I^{(L)} = \sum_k \left(t_{kL} c_{kL}^\dagger d_A + h.c.\right)$$

Fermionen ↓, Bosonen ↓

$$H_I^{(B)} = \left(d_A^\dagger d_B + d_B^\dagger d_A\right) \otimes \sum_k \left(h_k b_k + h_k^* b_k^\dagger\right)$$

$\|00\rangle$	:	0	Annahme (oBdA)
$\|10\rangle$	:	ε_A	$\varepsilon_A < \varepsilon_B$
$\|01\rangle$	:	ε_B	
$\|11\rangle$	:	$\varepsilon_A + \varepsilon_B + U$	

$$\mathcal{L}_L^{pop} = \Gamma_L \begin{pmatrix} -f_L(\varepsilon_A) & 1-f_L(\varepsilon_A) & 0 & 0 \\ f_L(\varepsilon_A) & -[1-f_L(\varepsilon_A)] & 0 & 0 \\ 0 & 0 & -f_L(\varepsilon_A+U) & 1-f_L(\varepsilon_A+U) \\ 0 & 0 & f_L(\varepsilon_A+U) & -[1-f_L(\varepsilon_A+U)] \end{pmatrix}$$

$$\Gamma_\nu(\omega) \approx \Gamma_\nu$$

analog $\mathcal{L}_R^{pop}$

$$\mathcal{L}_B^{pop} = \Gamma \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -n_B(\varepsilon_B-\varepsilon_A) & 1+n_B(\varepsilon_B-\varepsilon_A) & 0 \\ 0 & n_B(\varepsilon_B-\varepsilon_A) & -[1+n_B(\varepsilon_B-\varepsilon_A)] & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$n_B(\varepsilon_B-\varepsilon_A) = \frac{1}{e^{\beta_B(\varepsilon_B-\varepsilon_A)}-1}$$

$$\mathcal{L}^{pop} = \mathcal{L}_L^{pop} + \mathcal{L}_R^{pop} + \mathcal{L}_B^{pop}$$

Bsp: Ströme im SS

$\dot{\bar{S}}_i \geq 0$

$$\dot{\bar{S}}_i = -\beta_B \bar{I}_E^{(B)} - \beta_L\left[\bar{I}_E^{(L)} - \mu_L \bar{I}_M^{(L)}\right] - \beta_R\left[\bar{I}_E^{(R)} - \mu_R \bar{I}_M^{(R)}\right] \geq 0$$

$$\bar{I}_M = \bar{I}_M^{(L)} = -\bar{I}_M^{(R)}$$

$$\bar{I}_E^{(L)} + \bar{I}_E^{(R)} + \bar{I}_E^{(B)} = 0$$

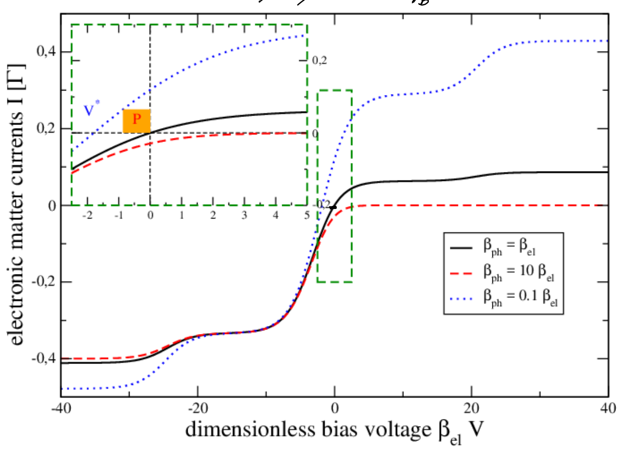
$$\rightarrow \dot{\bar{S}}_i = (\beta_R - \beta_B)\bar{I}_E^{(B)} + (\beta_R - \beta_L)\bar{I}_E^{(L)} + (\beta_L\mu_L - \beta_R\mu_R)\bar{I}_M^{(L)}$$

tight-coupling: $\bar{I}_E^{(B)} = (\varepsilon_B - \varepsilon_A) \cdot \bar{I}_M^{(L)}$

Fall: $\beta_R = \beta_L = \beta_{el} \neq \beta_B$

$$\rightarrow \dot{\bar{S}}_i = \underbrace{\left\{[\beta_{el} - \beta_B] \cdot (\varepsilon_B - \varepsilon_A) + \beta_{el}(\mu_L - \mu_R)\right\}}_{} \bar{I}_M^{(L)} \geq 0$$

bei $V^* = \mu_L^* - \mu_R^* = \left(\frac{T_{el}}{T_B} - 1\right)(\epsilon_B - \epsilon_A)$ verschwindet der Strom



$\bar{P} = -(\mu_L - \mu_R)\bar{I}_M^{(L)}$ $\qquad \eta = \dfrac{-(\mu_L - \mu_R)\cdot \bar{I}_M^{(L)} \cdot \Theta(P)}{\dot{Q}_{in}}$

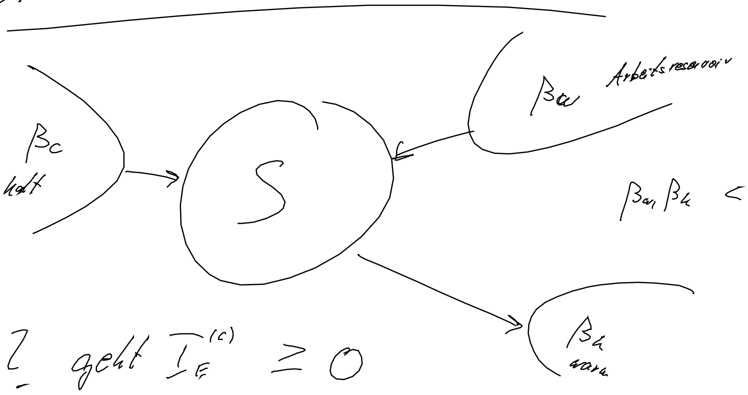
a.) $\eta_{T_B > T_{el}} \leq 1 - \dfrac{T_{el}}{T_B}$

b.) $\eta_{T_B < T_{el}} \leq 1 - \dfrac{T_B}{T_{el}}$

$\Big\} \leq \eta_{CA}$

$\dot{Q}_{in} = \dot{Q}^{(L)} + \dot{Q}^{(R)}$

2.8. Machbarkeit des Kühlens mit 3 Terminal-Systemen



$\beta_w, \beta_h < \beta_c$

2. geht $\bar{I}_E^{(c)} \geq 0$

2. HS: $\underbrace{\underbrace{(\beta_w - \beta_c)}_{\leq 0}\underbrace{\bar{I}_E^{(c)}}_{\geq 0}}_{\leq 0} + \underbrace{(\beta_w - \beta_h)\cdot \bar{I}_E^{(h)}}_{\geq 0} \geq 0$ $\qquad$ aus $\bar{I}_E^{(w)} = -\left(\bar{I}_E^{(c)} + \bar{I}_E^{(h)}\right)$

$\longrightarrow \beta_c > \beta_h > \beta_w$

Wann geht das?

1. Versuch: 2 levels $\qquad E_0 < E_1$

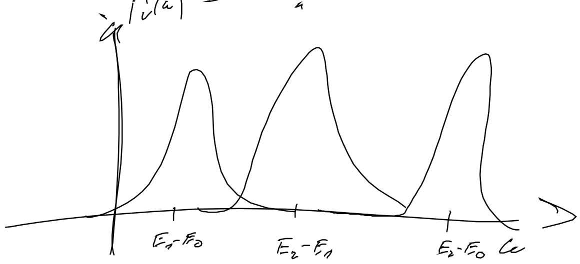
$$\mathcal{L}_2^{pop} = \Gamma_c \begin{pmatrix} -n_c & 1+n_c \\ n_c & -(1+n_c) \end{pmatrix} + \Gamma_h \begin{pmatrix} -n_h & 1+n_h \\ +n_h & -(1+n_h) \end{pmatrix} + \Gamma_w \begin{pmatrix} -n_w & 1+n_w \\ +n_w & -(1+n_w) \end{pmatrix}$$

$$n_\nu = \frac{1}{e^{\beta_\nu(E_1-E_0)}-1}$$

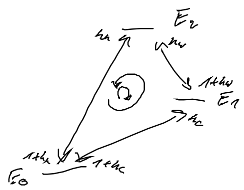
$$\rightsquigarrow \quad \bar{P}_0 = \frac{1+\bar{n}}{1+2\bar{n}}, \quad \bar{P}_1 = \frac{\bar{n}}{1+2\bar{n}} \; : \; \bar{n} = \frac{\Gamma_c \cdot n_c + \Gamma_h \cdot n_h + \Gamma_w \cdot n_w}{\Gamma_c + \Gamma_h + \Gamma_w} > n_c$$

$\rightsquigarrow$ kühlen geht nicht!

$$\Gamma_\nu(\omega) = 2\pi \sum_k |h_{k\nu}|^2 \delta(\omega - \omega_{k\nu})$$



2. Versuch: 3 level



$E_0 < E_1 < E_2$

$$\mathcal{L}_3^{pop} = \begin{pmatrix} -\Gamma_c \cdot n_c - \Gamma_h \cdot n_h & \Gamma_c(1+n_c) & \Gamma_h(1+n_h) \\ \Gamma_c \cdot n_c & -\Gamma_w \cdot n_w - \Gamma_c(1+n_c) & \Gamma_w(1+n_w) \\ \Gamma_h \cdot n_h & \Gamma_w \cdot n_w & -\Gamma_w(1+n_w) - \Gamma_h(1+n_h) \end{pmatrix}$$

$$n_h = \frac{1}{e^{\beta_h(E_2-E_0)}-1}$$

$$n_c = \frac{1}{e^{\beta_c(E_1-E_0)}-1}$$

$\rightsquigarrow$ $n_c > n_h$ ist möglich trotz $\beta_c > \beta_h$

$$I_E^{(c)} = \mathrm{Tr}\left\{ H_S \left(\mathcal{L}_3^{(c)} \bar{\rho} \right) \right\}$$

$$\mathcal{H}_S = \begin{pmatrix} E_0 & 0 & 0 \\ 0 & E_1 & 0 \\ 0 & 0 & E_2 \end{pmatrix}$$

$$\lim_{n_w \to \infty} \bar{I}_E^{(c)} = \underbrace{(E_1 - E_0)}_{\geq 0} \frac{\Gamma_c \cdot \Gamma_h (n_c - n_h)}{\Gamma_c(1+3n_c) + \Gamma_h(1+3n_h)}$$

geht wenn $\beta_h(E_2 - E_0) < \beta_c(E_1 - E_0)$

analog $\bar{I}_E^{(w)} = (E_2 - E_0) \left(\quad " \quad \right)$

$$COP = \frac{E_1 - E_0}{E_2 - E_0} \Theta(n_c - n_h) \leq \frac{T_c}{T_h - T_c} = COP_{Ca}$$