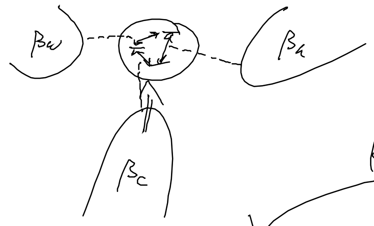
Fr. 5.7. $8^{00} - 10^{00}$ EB 301

Nachteilsausgleich 5.7. 8^{00} EW 705

<u>Wdh</u> • 3-Terminal Kältemaschine



$$\boxed{m\ddot{x} + \gamma\dot{x} + \frac{\partial V}{\partial x} = f_{stoch}(t)}$$

• Fokker-Planck-Gleichung <u>unterdämpft</u>

+ $\partial_t P = \left\{ \frac{\gamma}{m} - v\cdot\partial_x + \left(\frac{1}{m}\frac{\partial V}{\partial x} + \frac{\gamma}{m}\cdot v\right)\partial_v + D\,\partial_v^2 \right\} P(x,v,t)$

+ $\bar{P} \propto e^{-\beta\left[\frac{1}{2}mv^2 + V(x)\right]}$

+ $V(x) = \frac{1}{2}kx^2$

+ 1. HS: $\frac{d}{dt}\langle E\rangle = \frac{d}{dt}\left\langle \frac{1}{2}mv^2 + \frac{1}{2}kx^2\right\rangle = I_E(t)$

+ 2. HS: $\frac{d}{dt} S_{univ.} = \frac{d}{dt} S_{System} + \left(-\beta I_E(t)\right) \geq 0$

• analog überdämpft : $m\ddot{x} \approx 0$

4.2.3 Verbindung Ratengleichung

<u>überdämpft:</u> $x \longrightarrow x_i$ $\quad E_i = V(x_i) = V_i$

$P(x,t) \longrightarrow P(x_i,t) = P_i(t)$

$\partial_x P(x,t) \longrightarrow \frac{P_{i+1}(t) - P_{i-1}(t)}{2\Delta x} \rightsquigarrow \frac{P_{i+1/2}(t) - P_{i-1/2}(t)}{\Delta x}$ $\quad \left(\frac{P(x+\frac{\Delta x}{2}) - P(x-\frac{\Delta x}{2})}{\Delta x}\right)$

$\dot{P}_i = \sum_j W_{ij}\cdot P_j$ $\quad \Delta V = V_{i+1} - V_i$

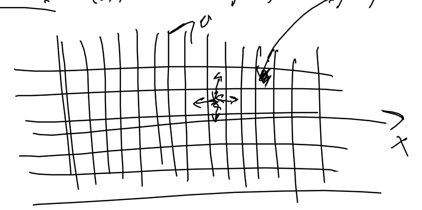
$\partial_x^2 P(x,t) \longrightarrow \frac{P_{i+1}(t) - 2P_i(t) + P_{i-1}(t)}{\Delta x^2}$

$\frac{W_{i,i+1}}{W_{i+1,i}} = 1 + \frac{\Delta V}{k_B T} + \mathcal{O}(\Delta V^2) = e^{\beta\Delta V} + \mathcal{O}(\Delta V^2)$

$\partial_x\left[\partial_x P(x)\right] = \frac{P'(x+\frac{\Delta x}{2}) - P'(x-\frac{\Delta x}{2})}{\Delta x}$

$= \frac{P(x+\Delta x) - P(x) - \left[P(x) - P(x-\Delta x)\right]}{\Delta x^2}$

<u>unterdämpft:</u> $P(x,v) \to P(x_i,v_j,t) = P_{ij}(t)$



4.3. Quanten-Otto-Prozess

• klassisch: 2 Adiabaten & 2 Isochoren

• jetzt HO

$$H(t) = \frac{p^2}{2m} + \frac{1}{2} m\,\omega(t)^2 x^2 \qquad x = \sqrt{\frac{\hbar}{2m\omega(t)}}\left[a^\dagger(t) + a(t)\right]$$

$$= \hbar\omega(t)\left(a^\dagger(t)a(t) + 1/2\right) \qquad p = i\sqrt{\frac{m\hbar\omega(t)}{2}}\left[a^\dagger(t) - a(t)\right]$$

$$\left[a(t), a^\dagger(t')\right] = \frac{1}{2}\left[\sqrt{\frac{\omega(t)}{\omega(t')}} + \sqrt{\frac{\omega(t')}{\omega(t)}}\right] \rightsquigarrow \left[H(t), H(t')\right] \neq 0$$

• $|\dot\psi\rangle = -\frac{i}{\hbar} H(t)\,|\psi\rangle$

+ falls $[H(t), H(t')] = 0 \rightsquigarrow U = \exp\left\{-\frac{i}{\hbar}\int^t H(t')dt'\right\}$

+ $H(t) = H(t+T) \rightsquigarrow U(t) = \underbrace{K(t)}_{K(t+T)} e^{-\frac{i}{\hbar}t\Omega}$ "Floquet-Theorie"

+ falls $\frac{\langle m(t)|\dot H|n(t)\rangle}{\left[E_n(t)-E_m(t)\right]^2} \ll 1$ ($n \neq m$) $\qquad H(t)\,|n(t)\rangle = E_n(t)\,|n(t)\rangle$

$\rightsquigarrow$ "adiabatische Näherung"

$$\rightsquigarrow U_{ad}(t) = \sum_n e^{-i\gamma_n(t)}\,|n(t)\rangle\langle n(0)|$$

4.3.1. Unitäre Prozessschritte

$$\dot\rho = -\frac{i}{\hbar}\left[H(t), \rho\right] \qquad \rho(0) = \sum_n P_n^0\,|n(0)\rangle\langle n(0)|$$

$$\rightsquigarrow \rho(t) = \sum_n P_n^0\,|n(t)\rangle\langle n(t)| = U_{ad}(t)\,\rho(0)\,U_{ad}^\dagger(t)$$

WS bleiben gleich $\rightsquigarrow$ auch die Entropie bleibt gleich

+ von Neumann Entropie

$$S(\rho) = S(U\rho U^\dagger) \qquad \text{immer:} \quad \frac{d}{dt}S = -\mathrm{Tr}\left\{\dot\rho \ln\rho + \cancel{\rho\,\rho^{-1}\cdot\dot\rho}\right\} = +\mathrm{Tr}\left\{\frac{i}{\hbar}\left[H(t),\rho\right]\ln\rho\right\} = 0$$

4.3.2. Dissipative Prozessschritte

Lindblad ME
$$\dot\rho = -\frac{i}{\hbar}\left[H, \rho\right] + \Gamma(1+n_B)\left[a\rho a^\dagger - \frac{1}{2}a^\dagger a\rho - \frac{1}{2}\rho a^\dagger a\right] + \Gamma n_B\left[a^\dagger\rho a - \frac{1}{2}aa^\dagger\rho - \frac{1}{2}\rho a a^\dagger\right] = \mathcal{L}\rho$$

$$\dot S \neq 0$$

Kopplungsstärke Reservoir

$$n_B = \frac{1}{e^{\beta\omega} - 1}$$

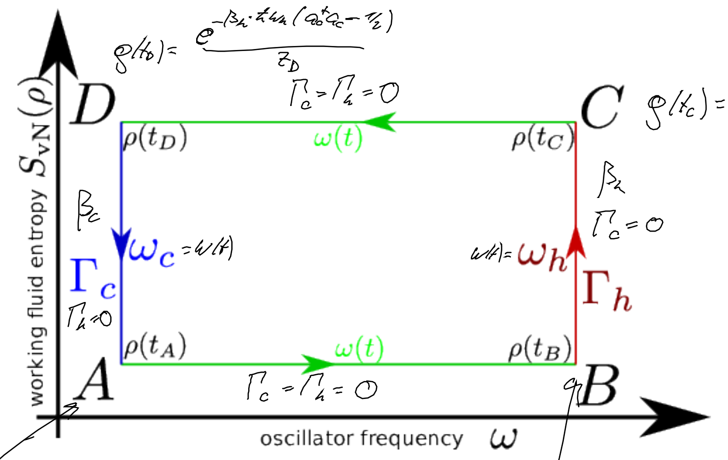
man zeigt: $\mathcal{L}\,e^{-\beta H} = 0$

$$\rho_c = \frac{e^{-\beta H}}{\mathrm{Tr}\{e^{-\beta H}\}} = \sum_n P_n\,|n\rangle\langle n| \qquad P_n = \frac{e^{-\beta E_n}}{\sum_m e^{-\beta E_m}}$$

EZ von H

$$\frac{d}{dt}\langle n|\rho|n\rangle = \Gamma(1+n_B)\cdot(n+1)\,\langle n+1|\rho|n+1\rangle + \Gamma n_B\cdot n\,\langle n-1|\rho|n-1\rangle + \left[\ldots\right]\underbrace{\langle n|\rho|n\rangle}_{P_n}$$

4.3.3. Otto-Zyklus



$$\rho(t_D) = \frac{e^{-\beta_h \cdot \hbar\omega_h (a_D^\dagger a_c - 1/2)}}{Z_D}$$

$$\rho(t_C) = \frac{e^{-\beta_h \cdot \hbar\omega_h (a_h^\dagger a_h - 1/2)}}{Z_C}$$

$$\rho(t_A) = \frac{e^{-\beta_c \hbar\omega_c (a_c^\dagger a_c - 1/2)}}{Z_A}$$

$$\rho(t_B) = \frac{e^{-\beta_c \hbar\omega_c (a_h^\dagger a_h - 1/2)}}{\underbrace{Z_B}_{= Z_A}}$$

<u>Arbeit</u>: $\Delta W_{AB} = \mathrm{Tr}\{\hbar\omega_h (a_h^\dagger a_h + 1/2) \cdot \rho(t_B)\} - \mathrm{Tr}\{\hbar\omega_c (a_c^\dagger a_c + 1/2) \cdot \rho(t_A)\}$

$$= \frac{\hbar\omega_h - \hbar\omega_c}{e^{\beta_c \hbar\omega_c} - 1} + \frac{\hbar(\omega_h - \omega_c)}{2}$$

$$\Delta W_{BC} = \ldots = \frac{\hbar\omega_c - \hbar\omega_h}{e^{\beta_h \cdot \hbar\omega_h} - 1} + \frac{\hbar(\omega_c - \omega_h)}{2}$$

<u>Wärme</u> $\Delta Q_{BC} = \mathrm{Tr}\{\hbar\omega_h (a_h^\dagger a_h)[\rho(t_C) - \rho(t_B)]\} = \frac{\hbar\omega_h}{e^{\beta_h \hbar\omega_h} - 1} - \frac{\hbar\omega_h}{e^{\beta_c \hbar\omega_c} - 1}$

ΔQ_{DA} analog

$$\rightsquigarrow \Delta W_{ges} = -\Delta W_{AB} - \Delta W_{CD} = \hbar(\omega_h - \omega_c)\left[\frac{1}{e^{\beta_h \hbar\omega_h} - 1} - \frac{1}{e^{\beta_c \hbar\omega_c} - 1}\right]$$

$$\eta = \frac{\Delta W_{ges}}{\Delta Q_{BC}} \Theta(\Delta W_{ges}) = \left(1 - \frac{\omega_c}{\omega_h}\right) \cdot \Theta(\Delta W_{ges})$$

$\beta_h \cdot \omega_h \le \beta_c \cdot \omega_c$

$\omega_h^{max} = \frac{\beta_c}{\beta_h} \cdot \omega_c$

$$\eta_{max} = 1 - \frac{\beta_h}{\beta_c} = 1 - \frac{T_c}{T_h} = \underline{\underline{\eta_{Carnot}}}$$

realistischer: Effizienz bei ΔW_{ges}^{max}

$$\eta_{max}^{CA} = 1 - \sqrt{\frac{T_c}{T_h}} < \eta_{Carnot}$$

noch realistischer: numerisch lösen

Lindblad $\underbrace{\frac{d}{dt}\left(-\mathrm{Tr}\{\rho \ln \rho\}\right)}_{\frac{d}{dt} S_{SYS}} \underbrace{- \beta \mathrm{Tr}\{H \dot{\rho}\}}_{+ \frac{d}{d} S_{RES}} \ge 0$

Viel Erfolg bei der Klausur :)